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Performance Improvement of JSCC Scheme through Re-designing Channel Code

Qiwang Chen, Lin Wang, *Senior Member, IEEE*, Shaohua Hong, Zixiang Xiong, *Fellow, IEEE*

***Abstract*—For joint source channel coding (JSCC) scheme-based on double protograph low-density parity-check (DP-LDPC) codes, due to joint iterative decoding between source and channel decoder, it is found that the optimal channel code in separate system is not optimal in joint system. In this letter, a novel joint protograph extrinsic information transfer (JPEXIT) analysis is proposed for the DP-LDPC system over AWGN and re-designing scheme for channel code is put forward to enhance performance of DP-LPDC system. Both the EXIT analysis and simulated results show performance improvement of proposed systems. This verifies the viewpoint that channel code should be re-designed for JSCC scheme even if optimal channel codes is selected over AWGN in separate systems.**

***Index Terms*—DP-LDPC, Joint PEXIT, JSCC, Performance improvement, Re-design**

I. Introduction

An innovative joint source-channel coding (JSCC) scheme where double low-density parity-check (D-LDPC) codes are used as source code and channel code is proposed in [1]. The redundancy left by source encoder is exploited by channel decoder to reduce the bit error rate (BER) through joint belief propagation (BP) algorithm. This is the same as other JSCC schemes [2][3].

Owing to high-speed encoding and linear decoding implementations, protograph LDPC (P-LDPC) code was introduced into this JSCC scheme to improve performance, named as double protograph low-density parity-check (DP-LDPC) system [4]. Recently, it was found that source statistics plays a more vital role in this JSCC scheme [5] and a matching criterion [6] between source statistics and source coding rate was presented, which facilitated the prediction of error-floor region performance below 10^{-6}. To further lower the error floor, an idea introduces *linking matrix* [7], e.g., inserting new edges between the check nodes (CNs) of channel code and the variable nodes (VNs) of the source code in tanner graph mentioned in [1], which can increase the amount of information about the source bits available at the decoder. All of the schemes mentioned above have similar optimization procedure that the source code is mainly optimized to lower the error floor when optimal channel code over AWGN in separate system is arranged as its channel code part.

Although the error floor has been solved effectively, the topic of improvement of water-fall region in this JSCC scheme is not studied so far. In this work, it is found that the optimal channel code in separate system is not optimal in joint system due to joint iterative decoding between source decoder and channel decoder. Thus, we propose a different optimization direction that P-LDPC code as channel code in joint system is re-designed to carry out whole system performance enhancement assuming that the error floor BER has satisfied wireless communication system. A novel joint protograph extrinsic information transfer (JPEXIT) analysis is proposed to calculate the decoding threshold of the joint protograph matrix including source code, channel code and two linking matrices from a global perspective. Moreover, a re-design scheme for channel code in the DP-LDPC system is put forward to enhance performance. Both the EXIT analysis and simulated results indicate that the DP-LDPC system with channel code re-designed show obvious improvement of water-fall region in comparison with the conventional DP-LDPC system. Therefore, re-designing channel code in JSCC scheme should be required to obtain performance improvement.

The letter is organized as follows. Section II introduces the system model. JPEXIT analysis and re-design scheme for channel code of the DP-LDPC system are presented in Section III. Simulation results are presented and discussed in Section IV. In section V, we draw our conclusions.

II. System Model

A protograph is a tanner graph with a relatively small number of nodes, defined as a base matrix $\mathbf{B} = (b_{i,j})$, where $b_{i,j}$ represents the number of edges connecting a VN v_j to a CN c_i. Then a corresponding parity-check matrix $\mathbf{H}$ of P-LDPC can be obtained by "copy-and-permute" operation on the base matrix.

For the DP-LDPC system, the tanner graph shown in Fig. 1, the joint base matrix with size $(m_{sc} + m_{cc}) \times (n_{sc} + n_{cc})$ is defined by

$$\mathbf{B}_J = \begin{bmatrix} \mathbf{B}_{sc} & \mathbf{B}_{L1} \\ \mathbf{B}_{L2} & \mathbf{B}_{cc} \end{bmatrix}, \quad (1)$$

where $\mathbf{B}_{sc}$ is a base matrix of source code with size $m_{sc} \times n_{sc}$, $\mathbf{B}_{cc}$ is a base matrix of channel code with size $m_{cc} \times n_{cc}$ and two linking code $\mathbf{B}_{L1}$ with size $m_{sc} \times n_{cc}$ and $\mathbf{B}_{L2}$ with size $m_{cc} \times n_{sc}$ represent the connectivity from CNs of source code to VNs of channel code and from VNs of source code to CNs

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Q. W. Chen, L. Wang and S. H. Hong are with the Department of Communication Engineering, Xiamen University, Xiamen, 361005, P. R. China. L. Wang is the corresponding author (e-mail: wanglin@xmu.edu.cn).

Z. Xiong is with the Department of Electrical and Computer Engineering, College of Engineering, Texas A&M University, College Station, TX 77843 USA(e-mail: zx@ece.tamu.edu)

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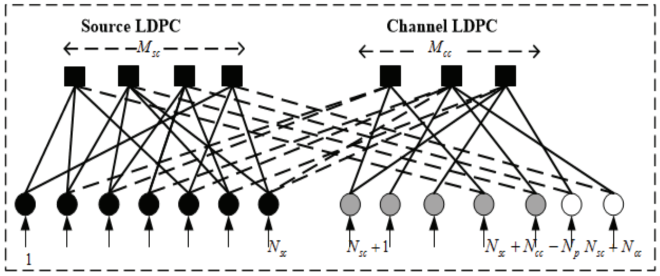

Fig. 1. The tanner graph of D-LDPC codes with inserting edges between CNs of channel code and VNs of source code

of channel code, respectively. Therefore, the joint parity-check matrix $\mathbf{H}_J$ of size $(M_{sc}+M_{cc})\times(N_{sc}+N_{cc})$ can be derived:

$$\mathbf{H}_J = \begin{bmatrix} \mathbf{H}_{sc} & \mathbf{H}_{L1} \\ \mathbf{H}_{L2} & \mathbf{H}_{cc} \end{bmatrix}, \tag{2}$$

where $\mathbf{H}_{sc}$ is source parity-check matrix with size $M_{sc}\times N_{sc}$, and $\mathbf{H}_{cc}$ is channel parity-check matrix with size $M_{cc}\times N_{cc}$. This letter mainly studies the punctured P-LDPC codes and the number of puncture nodes is N_p, where N_p is less than N_{sc}.

Let $\mathbf{s}$ be the source sequence that takes value from a binary i.i.d Bernoulli (p_v) source with entropy

$$H = -p_v \log_2 p_v - (1-p_v)\log_2(1-p_v), \tag{3}$$

where $p_v < 1/2$. In the encoding, for zero matrix $\mathbf{H}_{L2}$, the encoding process is a serial concatenation of two LDPC codes, e.g. $\mathbf{H}_{sc}$ and $\mathbf{H}_{cc}$, where $\mathbf{H}_{sc}$ performs source compression and $\mathbf{H}_{cc}$ performs error control coding. However, the performance of error floor may not satisfy the demand. To further improve the error floor, the non-zero matrix $\mathbf{H}_{L2}$ is considered [7], where a new channel coding matrix is constructed by the non-zero $\mathbf{H}_{L2}$ and $\mathbf{H}_{cc}$ and the corresponding information bits for channel coding are comprised of the original source sequence and the compressed bits generated by $\mathbf{H}_{sc}$. For more details about this discussion, the interested reader should refer to [7].

According to different initial log-likelihood ratios (LLR), all of the VNs are divided into three types, e.g. the source VNs, the transmitted channel VNs and the punctured channel VNs. In Fig. 1, these three types of the VNs are depicted as dark filled circles, gray filled circles and white filled circles, respectively. In the decoding, the joint BP algorithm is performed after the three types of the VNs are initialized mentioned in [1][4][5].

III. Analysis and Re-design of DP-LDPC Codes

A. Joint PEXIT algorithm

The PEXIT algorithm over Rayleigh fading channel has been mentioned in [6], where the procedure is comprised of two steps: channel decoder and source decoder. Moreover, it only analyzes the case with zero matrix $\mathbf{B}_{L2}$ but does not apply for the case with non-zero matrix $\mathbf{B}_{L2}$. In this section, a JPEXIT analysis from a global perspective is proposed to calculate the decoding threshold of joint base matrix $\mathbf{B}_J$, which can provide the error performance prediction in the following re-designing channel code in the DP-LDPC system. The detailed procedure is presented as follows.

Firstly, five types of mutual information (MI) are defined by

$I_{Ev}(i,j)$: the extrinsic MI from j-th VN to i-th CN
$I_{Ec}(i,j)$: the extrinsic MI from i-th CN to j-th VN
$I_{Av}(i,j)$: the a prior MI from j-th VN to i-th CN
$I_{Ac}(i,j)$: the a prior MI from i-th CN to j-th VN
$I_{APP}(j)$: the MI between a posterior LLR evaluated by j-th VN and the corresponding source bit s_j.

In addition, an indicator function is defined as follows:

$$\Phi(b_{i,j}) = \begin{cases} 1 & \text{if } b_(i,j) \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

and if a VN is punctured, its initial LLR value is 0. Moreover, $J(\sigma_{ch})$ represents the MI between a binary bit and its corresponding LLR value $L_{ch} \sim (\sigma_{ch}^2/2, \sigma_{ch}^2)$, given by [1]

$$J(\sigma_{ch}) = 1 - \int_{-\infty}^{\infty} \frac{e^{-(\xi-\sigma_{ch}^2/2)^2/2\sigma_{ch}^2}}{\sqrt{2\pi\sigma_{ch}^2}} \cdot log_2(1+e^{-\xi})d\xi. \tag{5}$$

Finally, the proposed JPEXIT algorithm for DP-LDPC over AWGN is described as follows.

1) The MI update from VNs to CNs

For $j = 1,\cdots,n_{sc}$ and $i = 1,\cdots,m_{sc}+m_{cc}$

$$\begin{aligned} I_{Ev}(i,j) =& \Phi(b_{i,j}) J_{BSC}(\sum_{s\neq i} b_{i,j}[J^{-1}(I_{Av}(s,j))]^2 \\ &+ (b_{i,j}-1)[J^{-1}(I_{Av}(i,j))]^2, p_v). \end{aligned} \tag{6}$$

The function J_{BSC} is defined as [1]

$$J_{BSC}(\mu, p_v) = (1-p_v) I(V;\chi^{(1-p_v)}) + p_v I(V;\chi^{p_v}),$$

where $I(V;\chi)$ is the MI between the VN of the source and χ, $\chi^{(1-p_v)} \sim N(\mu + Z_v^{sc}, 2\mu)$ and $\chi^{p_v} \sim N(\mu - Z_v^{sc}, 2\mu)$ with $Z_v^{sc} = \ln((1-p_v)/p_v)$.

For $j = n_{sc}+1,\cdots,n_{sc}+n_{cc}$ and $i = 1,\cdots,m_{sc}+m_{cc}$

$$\begin{aligned} I_{Ev}(i,j) =& \Phi(b_{i,j}) J\Bigg(\sqrt{\sum_{s\neq i} b_{s,j}[J^{-1}(I_{Av}(s,j))]^2} \\ &\overline{+(b_{i,j}-1)[J^{-1}(I_{Av}(i,j))]^2 + \sigma_{ch,j}^2}\Bigg). \end{aligned} \tag{7}$$

For $j = 1,\cdots,n_{sc}+n_{cc}$ and $i = 1,\cdots,m_{sc}+m_{cc}$

$$I_{Ac}(i,j) = I_{Ev}(i,j).$$

2) The MI update from CNs to VNs

For $j = 1,\cdots,n_{sc}+n_{cc}$ and $i = 1,\cdots,m_{sc}+m_{cc}$

$$\begin{aligned} I_{Ec}(i,j) =& \Phi(b_{i,j}) \Bigg(1 - J\sqrt{(\sum_{s\neq i} b_{s,j}[J^{-1}(1-I_{Ac}(i,s))]^2)} \\ &\overline{+(b_{i,j}-1)[J^{-1}(1-I_{Ac}(i,j))]^2}\Bigg) \end{aligned} \tag{8}$$

and set $I_{Av}(i,j) = I_{Ec}(i,j)$

3) The APP-LLR MI evaluation

For $j = 1,\cdots,n_{sc}$ and $i = 1,\cdots,m_{sc}$

$$I_{APP}(j) = J_{BSC}(\mu(j), p) \tag{9}$$

where $\mu(j) = \sum_i b_{i,j}[J^{-1}(I_{Av}(i,j))]^2$.

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TABLE I
DECODING THRESHOLDS OF DIFFERENT CHANNEL CODES IN THE DP-LDPC SYSTEM FOR DIFFERENT SOURCE STATISTICS WHEN SOURCE CODE IS R4JA, THE $\mathbf{B}_{L1}$ IS BEST CONNECTIVITY AND THE $\mathbf{B}_{L2}$ IS A ZERO MATRIX, E.G. EQUATION (10).

	$p_v = 0.01$	$p_v = 0.015$	$p_v = 0.02$
$\mathbf{B}_{AR4JA}$	-2.524 dB	-1.450 dB	-0.632 dB
$\mathbf{B}_{M1}$	-3.015 dB	-1.797 dB	-0.931 dB
$\mathbf{B}_{M2}(\mathbf{B}_{IARA-1})$	-3.145 dB	-1.984 dB	-1.155 dB
$\mathbf{B}_{AR3A}$	-3.248 dB	-1.910 dB	-0.965 dB
$\mathbf{B}_{IARA-2}$	-3.438 dB	-2.254 dB	-1.379 dB

The iteration of MI stops until $I_{APP}(j) = 1$.

Note that the proposed JPEXIT algorithm updates the MI between CNs and VNs as a single tanner graph. The convergence behavior of different $\mathbf{B}_{L1}$ and $\mathbf{B}_{L2}$ can be discussed and analyzed easily. The best $\mathbf{B}_{L1}$ is that the VNs of higher degree are assigned to the compressed bits. In order to depict the design of $\mathbf{B}_{L1}$ clearly, we consider R4JA codes (1/4-coderate) [10] as source code and AR4JA codes (1/2-coderate) [10] as channel code as follows. When the $\mathbf{B}_{L2}$ is zero matrix, the joint base $\mathbf{B}_J$ with best convergence behavior is given by

$$\mathbf{B}_J = \left[\begin{array}{cccccccc|ccccc} 3 & 1 & 3 & 1 & 3 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 3 & 1 & 3 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 1 \end{array}\right]. \quad (10)$$

B. A Re-design Scheme for P-LDPC Channel Codes in JSCC

It is well known that a good protograph (a low decoding threshold) contains one or more degree-1 VNs (a pre-coder), one very high degree VN and several degree-2 VNs. Moreover, linear minimum distance growth exhibiting low error floor requires the maximum number of degree-2 VNs in the protograph to be limited by the total number of checks (expect for the pre-coder) minus 1[8]. Thus, the AR4JA code is considered as a good protograph with certain complexity (the maximum number of parallel edges is 3).

For separate system, the decoder is a simple P-LDPC decoder where the messages are exchanged iteratively between the CNs and the VNs of the channel code. Due to joint iterative decoding between source decoder and channel decoder in the DP-LDPC system, the LLR values are exchanged in each joint iterative process. The optimal LDPC channel code in separate system may not be optimal as channel code in joint system.

To verify the above conjecture, we increase the proportion of degree-2 VNs gradually using the modification of 1/2-coderate AR4JA as example. Two new protographs are proposed as follows.

$$\mathbf{B}_{M1} = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 3 & 1 & 1 & 1 \\ 0 & 2 & 2 & 1 & 1 \end{bmatrix} \quad (11)$$

and

$$\mathbf{B}_{M2} = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 3 & 1 & 1 & 1 \\ 0 & 3 & 1 & 1 & 1 \end{bmatrix} \quad (12)$$

where the number of degree-2 VNs is 2 and 3, respectively. With the number of degree-2 VNs increased, the lower decoding threshold is obtained, which is depicted in Table I for different source statistics.

Based on the aforementioned observation, **a re-design scheme for the P-LDPC channel codes** in DP-LDPC system is proposed as follows.

a) Increase the proportion of degree-2 VNs by removing some edges of VNs with degree greater than 2 except for the VN with largest degree.

b) For every removed edge, add an extra connection to the VN with the largest degree so as to keep the complexity (number of edges).

As we know, AR3A [8] P-LDPC code, shown in Eq. (13), possesses low decoding threshold in separate system over AWGN but the performance of error floor region is worse than that of AR4JA.

$$\mathbf{B}_{AR3A} = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 & 1 \end{bmatrix} \quad (13)$$

According to the re-design scheme mentioned above, this P-LDPC code can be re-designed, given by

$$\mathbf{B}_{IARA-2} = \begin{bmatrix} 1 & 3 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 \end{bmatrix}. \quad (14)$$

Therefore, two new 1/2-coderate protographs are re-designed for the DP-LDPC system, where the first one ($\mathbf{B}_{M2}$ mentioned above) is the improvement of AR4JA code, denoted by $\mathbf{B}_{IARA-1}$, and the second one is the improvement of AR3A code [8], denoted as $\mathbf{B}_{IARA-2}$. It can be found that both of two new protographs have a precoding structure, a VN with large degree and three degree-2 VNs. However, they are not with linear minimum distance growth and both of them can't converge in separate system analyzed by PEXIT algorithm in [11]. If we want to get the higher rate channel codes, the lengthening or the extension of LDPC code is discussed in [8]-[10] and the similar re-design scheme can be further proposed.

IV. PERFORMANCE COMPARISON

In this section, the performance of the AR4JA code, the AR3A code, the proposed IARA-1 code and IARA-2 code in JSCC scheme based on DP-LDPC codes is compared, where the source code is 1/4-coderate R4JA code and the code rate of the channel code is 1/2. The frame length is fixed at 3200 bits and the protograph is lifted by a factor of 800 using the progressive edge growth (PEG) algorithm [12] to obtain the parity-check matrix. The channel being considered is AWGN, the frame length is fixed at 3200 bits and the maximum decoding iteration is 50.

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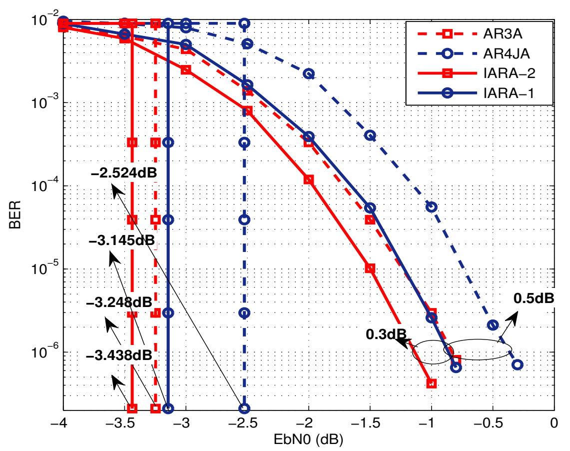

Fig. 2. The performance of different P-LDPC code as channel code in JSCC when $\mathbf{B}_{L2}$ is a zero matrix and $p_v = 0.01$

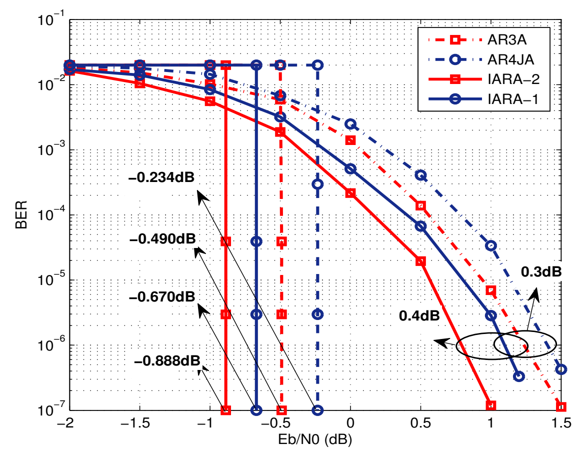

Fig. 3. The performance of different P-LDPC code as channel code in JSCC when $\mathbf{B}_{L2}$ contains two degree-4 VNs and $p_v = 0.02$

A. Zero Matrix $\mathbf{B}_{L2}$

For zero matrix $\mathbf{B}_{L2}$, when $p_v = 0.02$ and the code rate of source code is 1/4, the BER curves show error floor [1][4][5][6]. As shown in [5][6], when the source code is 1/4-coderate R4JA and $p_v = 0.01$, the source statistic is matched to the source coding rate and the problem of error floor is solved. Thus, the simulation results are given at $p_v = 0.01$ when $\mathbf{B}_{L2}$ is a zero-matrix.

As seen in Fig. 2, the practical performance of DP-LDPC system with proposed IARA-1 and IARA-2 channel codes have 0.5 dB and 0.3 dB coding gain compared with that of AR4JA and AR3A codes, respectively, which are in accord with the analysis of decoding thresholds.

B. Non-zero matrix $\mathbf{B}_{L2}$

If the source statistic is fixed at $p_v = 0.02$ or becomes greater, the method being free of error floor [7] is to introduce the non-zero matrix $\mathbf{B}_{L2}$. Therefore, when $p_v = 0.02$, the linking matrix $\mathbf{B}_{L2}$ containing two degree-4 VNs and six degree-0 VNs is given by

$$\mathbf{B}_{L2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 \end{bmatrix}. \qquad (15)$$

It is seen from Fig.3 that the IARA-1 and IARA-2 code as channel code outperform the AR4JA and AR3A code by 0.3 dB and 0.4 dB, respectively, which are consistent with the analysis of the decoding thresholds.

V. Conclusion

In this letter, a re-design scheme different from the design demand in separate system is proposed for the DP-LDPC system over AWGN owing to joint iterative decoding between source decoder and channel decoder. Two improved P-LDPC codes with more degree-2 VNs are constructed. Although these improved codes are against the demand of linear minimum distance growth, both EXIT analysis and simulation results reveal consistently that the DP-LDPC system with the improved codes as channel code show obvious improvement of water-fall region in comparison with the conventional DP-LDPC system.

The discovery of re-designing channel code for the DP-LDPC system may enlighten other JSCC schemes to obtain performance improvement. How to find optimal channel code in JSCC scheme is still an open problem.

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