

# Performance of Joint Source-Channel Coding Based on Protograph LDPC Codes over Rayleigh Fading Channels

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**Abstract**—This letter presents a joint protograph extrinsic information transfer (PEXIT) analysis for joint source and channel coding (JSCC) with double protograph-based low-density parity-check (DP\_LDPC) codes over single-input multiple-output (SIMO) Rayleigh fading channels. The impacts of source statistics and receive antenna diversity on convergence performance for the JSCC are investigated, and we find that source sparsity plays a more vital role than diversity orders in improving both convergence and error performances under fading environments.

**Index Terms**—Joint source-channel coding (JSCC), joint PEXIT analysis, source statistics, receive antenna diversity, Rayleigh fading channels.

## I. INTRODUCTION

IN the presence of different approaches to joint source-channel coding (JSCC), an innovative structure that employs low-density parity-check (LDPC) codes as both source and channel codes [1], [2], referred to as double LDPC (D\_LDPC) codes, has been illustrated to outperform tandem coding structures. However, the LDPC codes utilized in [1] and [2] are regular and irregular, which suffer from high error-floor and computational complexity. Therefore, protograph-based LDPC codes [3] have been introduced into D\_LDPC systems (DP\_LDPC in the following) by [4], where the error performance can be improved significantly. Moreover, the DP\_LDPC codes have also been employed to transmit images over additive white Gaussian noise (AWGN) channels [5], and simulation results demonstrated that a reduction of channel signal-to-noise ratio (SNR) can be obtained while a large peak SNR (PSNR) gain can also be achieved.

Despite the fact that DP\_LDPC codes perform well over AWGN channel, it may not work well in a fading environment. Although protograph extrinsic information transfer (PEXIT) algorithms [6] have been further tailored to predict the decoding thresholds of protograph-based codes over single-input multiple-output (SIMO) fading channels [7], to the best of our knowledge, little is known on the analytical performance for DP\_LDPC codes considering both source statistics and multiple receive antennas under fading conditions.

In this letter, we present a joint PEXIT analysis for JSCC using DP\_LDPC codes over Rayleigh fading channels, and the

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proposed algorithm concentrates on receive antenna diversity and source statistics. We then analyze the decoding thresholds of DP\_LDPC codes to predict the error performance in the waterfall region. Simulation results show that decoding thresholds and error performance are more sensitive to source sparsity than diversity orders. Therefore, sparse representation of sources should be more concerned to obtain good decoding convergence behavior for JSCC systems in fading situations.

## II. SYSTEM MODEL

Considering that Markov source structures may be limited in many applications scenarios, such as speech recognition, image segmentation, etc [8]. This paper focuses on the more general case of hidden Markov model (HMM), where the state of the model is related to the output probabilistically as opposed to deterministically. We consider an HMM with  $S = 2$  states can be defined by the vector  $[a_{00}, a_{11}, b_{00}, b_{11}]$ , where  $a_{ij} = P_i(S_j|S_i)$  denotes the probability of transition from state  $S_i$  to  $S_j$ ,  $0 \leq i, j \leq S - 1$ , and  $b_{ie} = P_o(e|S_i)$  represents the probability of getting the binary output  $e$  in state  $S_i$ . Moreover,  $P_1$  is defined as the stationary probability of the symbols 1.

Let  $\mathbf{w} = w_1 w_2 \dots$  be the source sequence that takes values  $w_k \in \{0, 1\}$  and is generated by an HMM. Then the source sequence is compressed by an unpunctured protograph LDPC code into binary sequence  $\mathbf{b}$ , followed by another protograph LDPC code to generate channel codeword  $\mathbf{c}$ . Subsequently, the codeword is passed to a binary-phase-shift-keying (BPSK) modulator, and the output of which is denoted by  $\mathbf{x}$ . The modulated signal is further sent to the SIMO Rayleigh fading channel with one transmit antenna and  $N_r$  receive antennas.

Let  $\mathbf{h}$  be the channel vector of size  $N_r \times 1$ , whose entries are independent, complex, zero-mean, Gaussian variables with independent real and imaginary components having the same variance  $1/2$ . We define  $\mathbf{n}$  as a  $N_r \times 1$  complex Gaussian background noise with mean zero and covariance matrix  $\sigma_n^2 \mathbf{I}$ , where  $\mathbf{I}$  represents the  $N_r \times N_r$  identity matrix. Then, the receiver sees  $N_r \times 1$  signal vectors

$$\mathbf{y} = \mathbf{h}\mathbf{x} + \mathbf{n} \quad (1)$$

Assuming that  $\mathbf{h}$  is perfectly known to the receiver, and it changes for every symbol  $\mathbf{x}$  to satisfy ergodicity, and then the capacity of an ergodic SIMO channel is further given by [9]

$$H(\mathbf{w})R_c < C = E \left( \log_2 \det \left( \mathbf{I} + \frac{E_s}{N_0} \mathbf{h}\mathbf{h}^\dagger \right) \right), \quad (2)$$

$$H(\mathbf{w}) \leq R_s$$

where  $H(\mathbf{w})$  is the source entropy,  $E_s$  denotes the average energy per transmitted symbol, and  $\mathbf{h}^\dagger$  is the complex-conjugate

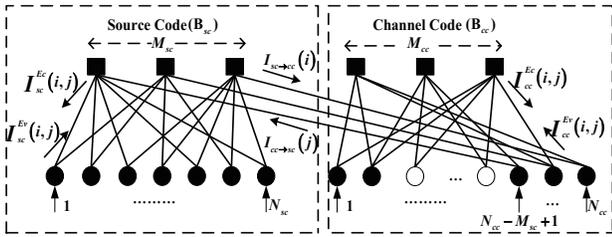


Fig. 1. Tanner graph representing the JSC decoder. The variable nodes connected to the channel are depicted with dark circles while check nodes are those dark squares, and blank circles denote the punctured variable nodes.

transpose of  $\mathbf{h}$ .  $R_s$  and  $R_c$  represent source code and channel code rates, respectively.

At the receiving terminal, the maximal-ratio combining (MRC) method is utilized to combine corrupted signals, and the output of which can be computed as  $z_j = \sum_{r=1}^{N_r} h_j^*[r]y_j[r]$ , where  $j$  indicates the coded bit number and  $r$  ( $r = 1 \cdots N_r$ ) marks the receive antenna number, \*denotes the complex conjugate. The joint source and channel (JSC) decoder [4] is further implemented to decode the combined signals. Moreover, the definition of  $E_b/N_0$  ( $E_b$  is the average energy per information bit) is expressed as

$$\frac{E_b}{N_0} = \frac{(N_r/R_c)E_s}{N_0} \quad (3)$$

In this letter, we assume that  $E_s$  is normalized to be 1, and then combining it with  $N_0 = 2\sigma_n^2$  gives

$$\sigma_n^2 = \frac{N_r}{2R_c(E_b/N_0)} \quad (4)$$

### III. JOINT PEXIT ALGORITHM FOR DP\_LDPC CODES

#### A. Assumptions of joint PEXIT algorithm

We first assume that the all-zero codeword is transmitted. Moreover, if a fixed channel vector  $\mathbf{h}_j$  is given for the  $j$ th coded bit, the associated initial channel log-likelihood ratio (LLR) with MRC method has been proved to follow a symmetric complex Gaussian distribution, and it can be expressed as [7]

$$L_{ch,j} = \ln \left( \frac{P_r(c_j = 0|z_j, \mathbf{h}_j)}{P_r(c_j = 1|z_j, \mathbf{h}_j)} \right) \sim CN \left( \frac{2}{\sigma_n^2} f_j, \frac{4}{\sigma_n^2} f_j \right). \quad (5)$$

where  $c_j$  denotes the  $j$ th codeword bit mapped onto the vector symbol  $\mathbf{x}$ , and

$$f_j = \sum_{r=1}^{N_r} |h_j[r]|^2. \quad (6)$$

is defined as channel factor. Thus, the joint PEXIT algorithm can be analyzed over SIMO fading channels with this property.

#### B. Joint PEXIT algorithm for DP\_LDPC codes

Fig.1 shows the Tanner graph of the JSC decoder, where the mutual information (MI) flow is also illustrated. A protograph with  $N$  variable nodes and  $M$  check nodes can be denoted by a base matrix  $\mathbf{B}$  of dimension  $M \times N$ . Accordingly,  $\mathbf{B}_{cc}$  with dimension  $M_{cc} \times N_{cc}$  and  $\mathbf{B}_{sc}$  with size of  $M_{sc} \times N_{sc}$  represent the base matrices of channel code and source code, respectively. Besides, an assumption is made that the last

$M_{sc}$  variable nodes in  $\mathbf{B}_{cc}$  are connected to the check nodes in  $\mathbf{B}_{sc}$ , and each check node is connected to only a single variable node. The  $(i, j)$ th element of  $\mathbf{B}_{cc}(\mathbf{B}_{sc})$ , denoted by  $b_{cc}^{i,j}(b_{sc}^{i,j})$ , represents the number of edges connecting the variable node  $v_j$  to check node  $c_i$ . Moreover, seven types of MI are defined as follows (with subscripts  $sc$  and  $cc$  indicating the source code  $\mathbf{B}_{sc}$  and channel code  $\mathbf{B}_{cc}$ , respectively):

- 1)  $I_{sc(cc)}^{A_v}(i, j)$ : the a priori MI between input LLR of  $v_j$  on each of the  $b_{sc(cc)}^{i,j}$  edges and the associated codeword bit  $v_j$ .
- 2)  $I_{sc(cc)}^{A_c}(i, j)$ : the a priori MI between input LLR of  $c_i$  on each of the  $b_{sc(cc)}^{i,j}$  edges and the associated codeword bit  $v_j$ .
- 3)  $I_{sc(cc)}^{E_v}(i, j)$ : the extrinsic MI between the LLR sent by  $v_j$  to  $c_i$  and the corresponding codeword bit  $v_j$ .
- 4)  $I_{sc(cc)}^{E_c}(i, j)$ : the extrinsic MI between the LLR sent by  $c_i$  to  $v_j$  and the corresponding codeword bit  $v_j$ .
- 5)  $I_{cc \rightarrow sc}(j)$ : the extrinsic MI between the LLR sent by  $v_j$  in  $\mathbf{B}_{cc}$  to  $c_i$  in  $\mathbf{B}_{sc}$  and the associated codeword bit, on each of the last  $m_{sc}$  edges connecting  $v_j$  in  $\mathbf{B}_{cc}$  to  $c_i$  in  $\mathbf{B}_{sc}$ .
- 6)  $I_{sc \rightarrow cc}(i)$ : the extrinsic MI between the LLR sent by  $c_i$  in  $\mathbf{B}_{sc}$  to  $v_j$  in  $\mathbf{B}_{cc}$  and the associated codeword bit, on each of the edges connecting  $c_i$  in  $\mathbf{B}_{sc}$  to  $v_j$  in  $\mathbf{B}_{cc}$ .
- 7)  $I_{sc}^{APP}(j)$ : the MI between a posteriori LLR evaluated by  $v_j$  in  $\mathbf{B}_{sc}$  and the associated codeword bit  $w_j$ .

In addition, the punctured label  $P_j$  of a variable node  $v_j$  is defined to be 0 if  $v_j$  is punctured and 1 otherwise.

We denote with  $J(\sigma_{ch})$  the MI between a codeword bit and its corresponding LLR value  $L_{ch} \sim N(\sigma_{ch}^2/2, \sigma_{ch}^2)$ , and it is given by [9]

$$J(\sigma_{ch}) = 1 - \int_{-\infty}^{\infty} \frac{e^{-(\xi - \sigma_{ch}^2/2)^2/2\sigma_{ch}^2}}{\sqrt{2\pi\sigma_{ch}^2}} \cdot \log_2(1 + e^{-\xi}) d\xi \quad (7)$$

Then, the joint PEXIT algorithm for DP\_LDPC codes over SIMO Rayleigh fading channels is described as follows:

#### 1) Channel Decoder

##### · Channel factors realization

Suppose we are given a channel vector  $\mathbf{h} = [h[1], \dots, h[N_r]]^T$  and the number of blocks of channel factors (denoted by  $K$ ), a matrix  $\mathbf{F} = (f_{k,j}) = \left( \sum_{r=1}^{N_r} |h_{k,j}[r]|^2 \right)$  of dimension  $K \times N_{cc}$  is generated to represent the  $K$  blocks of channel factors, with each row in  $\mathbf{F}$  indicating a group of channel factors for the  $N_{cc}$  variable nodes of channel codes.

##### · Initialization

Considering the punctured label  $P_j$  and the channel factor  $f_{k,j}$  ( $j = 1, 2 \cdots N_{cc}$  and  $k = 1, 2 \cdots K$ ), the corresponding variance of initial LLR value ( $\sigma_{ch,k,j}^2$ ) can be given by [7]

$$\sigma_{ch,k,j}^2 = \frac{4P_j f_{k,j}}{\sigma_n^2} = \frac{8R_c P_j f_{k,j}}{N_r} 10^{((E_b/N_0)/10)} \quad (8)$$

where  $E_b/N_0$  is in dB herein.

##### · Variable nodes to check nodes update

For  $j = 1 \cdots N_{cc} - M_{sc}$  and  $i = 1 \cdots M_{cc}$ , if  $b_{cc}^{i,j} \neq 0$ ,

$$I_{cc}^{E_v,k}(i, j) = J \left( \sqrt{\sum_{s \neq i} b_{cc}^{s,j} [J^{-1}(I_{cc}^{A_v}(s, j))]^2 + (b_{cc}^{i,j} - 1) [J^{-1}(I_{cc}^{A_v}(i, j))]^2 + \sigma_{ch,k,j}^2} \right).$$

For  $j = N_{cc} - M_{sc} + 1 \cdots N_{cc}$  and  $i = 1 \cdots M_{cc}$ , if  $b_{cc}^{i,j} \neq 0$ ,

$$I_{cc}^{E_{v,k}}(i, j) = \frac{J \left( \sqrt{\sum_{s \neq i} b_{cc}^{s,j} [J^{-1}(I_{cc}^{A_v}(s, j))]^2 + (b_{cc}^{i,j} - 1)[J^{-1}(I_{cc}^{A_v}(i, j))]^2} + \sigma_{ch,k,j}^2 + [J^{-1}(I_{sc \rightarrow cc}(j - (N_{cc} - M_{sc})))]^2 \right)}{+ [J^{-1}(I_{sc \rightarrow cc}(j - (N_{cc} - M_{sc})))]^2}.$$

If  $b_{cc}^{i,j} = 0$ ,  $I_{cc}^{E_{v,k}}(i, j) = 0$ .

• For  $j = 1 \cdots N_{cc}$  and  $i = 1 \cdots M_{cc}$ , we obtain the average MI of  $I_{cc}^{E_{v,k}}(i, j)$  by

$$E[I_{cc}^{E_{v,k}}(i, j)] = \frac{1}{K} \sum_{k=1}^K I_{cc}^{E_{v,k}}(i, j) \quad (9)$$

and set  $I_{cc}^{A_c}(i, j) = E[I_{cc}^{E_{v,k}}(i, j)]$ .

• **Check nodes to variable nodes update**

For  $j = 1 \cdots N_{cc}$  and  $i = 1 \cdots M_{cc}$ , if  $b_{cc}^{i,j} \neq 0$ ,

$$I_{cc}^{E_c}(i, j) = 1 - J \left( \frac{\sqrt{\sum_{s \neq j} b_{cc}^{i,s} [J^{-1}(1 - I_{cc}^{A_c}(i, s))]^2} + (b_{cc}^{i,j} - 1)[J^{-1}(1 - I_{cc}^{A_c}(i, j))]^2}{+ [J^{-1}(1 - I_{cc}^{A_c}(i, j))]^2} \right).$$

Set  $I_{cc}^{A_v}(i, j) = I_{cc}^{E_c}(i, j)$ ; and if  $b_{cc}^{i,j} = 0$ ,  $I_{cc}^{E_c}(i, j) = 0$ .

• **Variable nodes in  $\mathbf{B}_{cc}$  to check nodes in  $\mathbf{B}_{sc}$  update**

For  $j = N_{cc} - M_{sc} + 1 \cdots N_{cc}$  and  $i = 1 \cdots M_{cc}$ , if  $b_{cc}^{i,j} \neq 0$ ,

$$I_{cc \rightarrow sc,k}(j - (N_{cc} - M_{sc})) = \frac{J \left( \sqrt{\sum_i b_{cc}^{i,j} [J^{-1}(I_{cc}^{A_v}(i, j))]^2} + \sigma_{ch,k,j}^2 \right)}{+ [J^{-1}(I_{cc}^{A_v}(i, j))]^2}.$$

Then the MI is further averaged by

$$I_{cc \rightarrow sc}(j - (N_{cc} - M_{sc})) = \frac{1}{K} \sum_{k=1}^K I_{cc \rightarrow sc,k}(j - (N_{cc} - M_{sc})). \quad (10)$$

## 2) Source Decoder

• **Initialization**

A “virtual” channel corresponding to the source with noise distribution identical to the source statistics can be modeled by a binary symmetric channel (BSC) with crossover probability denoted by  $P_1$ . The probability density function of the LLR output from the equivalent channel is expressed as  $P_1 \delta(\psi + L_{sc}) + (1 - P_1) \delta(\psi - L_{sc})$ ,  $L_{sc} = \ln((1 - P_1)/P_1)$ . Then we denote with  $J_{BSC}(\cdot)$  a manipulation of the function  $J(\cdot)$  to take source statistics  $P_1$  into account, and it is given by [2]

$$J_{BSC}(\mu, P_1) = (1 - P_1)I(V; \chi^{(1-P_1)}) + P_1I(V; \chi^{P_1}). \quad (11)$$

where  $\chi^{(1-P_1)} \sim N(\mu + L_{sc}, 2\mu)$ ,  $\chi^{(P_1)} \sim N(\mu - L_{sc}, 2\mu)$ ;  $I(V; \chi)$  is the MI between LLR value of  $v_j$  in  $\mathbf{B}_{sc}$  and  $\chi$ .  $\mu$  represents the average LLR value obtained by variable node  $v_j$  in  $\mathbf{B}_{sc}$ .

• **Variable nodes to check nodes update**

For  $j = 1 \cdots M_{sc}$  and  $i = 1 \cdots N_{sc}$ , if  $b_{sc}^{i,j} \neq 0$ ,

$$I_{sc}^{E_v}(i, j) = J_{BSC} \left( \sum_{s \neq i} b_{sc}^{s,j} [J^{-1}(I_{sc}^{A_v}(s, j))]^2 + (b_{sc}^{i,j} - 1)[J^{-1}(I_{sc}^{A_v}(i, j))]^2, P_1 \right).$$

Set  $I_{sc}^{A_c}(i, j) = I_{sc}^{E_v}(i, j)$ ; and if  $b_{sc}^{i,j} = 0$ ,  $I_{sc}^{E_v}(i, j) = 0$ .

• **Check nodes to variable nodes update**

For  $j = 1 \cdots N_{sc}$  and  $i = 1 \cdots M_{sc}$ , if  $b_{sc}^{i,j} \neq 0$ ,

$$I_{sc}^{E_c}(i, j) = 1 - J \left( \frac{\sqrt{\sum_{s \neq j} b_{sc}^{i,s} [J^{-1}(1 - I_{sc}^{A_c}(i, s))]^2} + (b_{sc}^{i,j} - 1)[J^{-1}(1 - I_{sc}^{A_c}(i, j))]^2 + [J^{-1}(1 - I_{cc \rightarrow sc}(i))]^2}{+ [J^{-1}(1 - I_{cc \rightarrow sc}(i))]^2} \right).$$

Set  $I_{sc}^{A_v}(i, j) = I_{sc}^{E_c}(i, j)$ ; and if  $b_{sc}^{i,j} = 0$ ,  $I_{sc}^{E_c}(i, j) = 0$ .

• **Check nodes in  $\mathbf{B}_{sc}$  to variable nodes in  $\mathbf{B}_{cc}$  update**

For  $j = 1 \cdots N_{sc}$  and  $i = 1 \cdots M_{sc}$ , if  $b_{sc}^{i,j} \neq 0$ ,

$$I_{sc \rightarrow cc}(i) = 1 - J \left( \frac{\sqrt{\sum_j b_{sc}^{i,j} [J^{-1}(1 - I_{sc}^{A_c}(i, j))]^2}}{+ [J^{-1}(1 - I_{sc}^{A_c}(i, j))]^2} \right).$$

• **APP-LLR mutual information evaluation**

For  $j = 1 \cdots N_{sc}$  and  $i = 1 \cdots M_{sc}$ , if  $b_{sc}^{i,j} \neq 0$ ,

$$\mu(j) = \sum_i b_{sc}^{i,j} [J^{-1}(I_{sc}^{A_v}(i, j))]^2,$$

$$I_{sc}^{APP}(j) = J_{BSC}(\mu(j), P_1). \quad (12)$$

• **Iterate until  $I_{sc}^{APP}(j) = 1, \forall j$ .**

Note that each of the output extrinsic MI sent by variable node  $v_j$  in  $\mathbf{B}_{cc}$  (i.e.  $I_{cc}^{E_{v,k}}(i, j)$  and  $I_{cc \rightarrow sc,k}(j)$ ) depends on both the initial channel LLR  $\sigma_{ch,k,j}^2$  and the type of code used. Therefore, we need to compute the average values ((9) and (10)) based on the  $K$  blocks of channel factors. Moreover,  $K$  should be generated sufficiently large to simulate the ergodic fading channel that implemented by independent realizations of  $\mathbf{h}$ . In this letter, we set  $K = 10^5$ .

## IV. SIMULATION RESULTS AND DISCUSSION

In this paper, the rate 1/2 accumulate-repeat-by-3-accumulate (AR3A) code is employed as channel code and repeat-by-4-jagged-accumulate (R4JA) code with rate 1/4 [10] is utilized as source code, whose corresponding base matrices (denoted by  $\mathbf{B}_{cc}^{AR3A}$  and  $\mathbf{B}_{sc}^{R4JA}$ , respectively) are given by

$$\mathbf{B}_{cc}^{AR3A} = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 & 1 \end{bmatrix} \quad (13)$$

$$\mathbf{B}_{sc}^{R4JA} = \begin{bmatrix} 3 & 1 & 3 & 1 & 3 & 1 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 & 3 & 1 & 2 \end{bmatrix} \quad (14)$$

Note that the variable nodes associated with the second columns in (13) are punctured. Moreover, we terminate the simulation after 800 bit errors are detected at each  $E_b/N_0$ , and the JSC decoder performs a maximum of 100 BP iterations for each frame.

TABLE I  
DECODING THRESHOLDS AND CAPACITY ( $E_b/N_0$ ) CONSIDERING SOURCE STATISTICS AND DIVERSITY ORDERS OVER SIMO FADING CHANNEL.

HMM	$P_1$	Entropy $H(\mathbf{w})$	Capacity $E_b/N_0$ (dB)		Decoding thresholds $E_b/N_0$ (dB)					
			$N_r = 1$	$N_r = 4$	$N_r = 1$	Gap $\Delta$	$N_r = 2$	$N_r = 3$	$N_r = 4$	Gap $\Delta$
[0.02, 0.02, 0.99, 0.01]	0.01	0.08	-1.47	-1.51	-0.31	1.16	-0.86	-1.06	-1.14	0.37
[0.03, 0.04, 0.98, 0.01]	0.015	0.11	-1.43	-1.48	0.48	1.91	-0.12	-0.34	-0.46	1.02
[0.02, 0.02, 0.98, 0.02]	0.02	0.14	-1.38	-1.45	1.12	2.50	0.4	0.14	0.04	1.49
[0.03, 0.03, 0.96, 0.01]	0.025	0.17	-1.36	-1.44	1.48	2.84	0.75	0.47	0.38	1.82

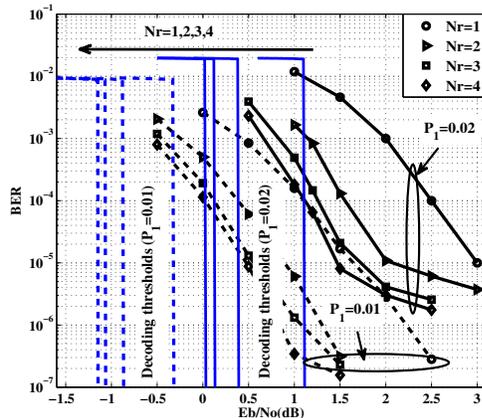


Fig. 2. Performance comparisons for source statistics  $P_1 = 0.01, 0.02$  with diversity orders  $N_r = 1, 2, 3, 4$  over Rayleigh fading channels.

In order to assess the system performance, we simulate the proposed joint PEXIT algorithm for sources generated by different HMMs, which result in different entropies. The frame length is fixed to 3200 bits.

The decoding thresholds and capacity at  $E_b/N_0$  of DP\_LDPC codes for different source statistics with diversity orders  $N_r = 1, 2, 3, 4$  are illustrated in Table. I (For our simulation, each channel formed by one transmit-receive antenna pair is faded independently and hence the diversity order equals  $N_r$ ). The results indicate that the decoding thresholds can be reduced either by making sources sparser (lower  $P_1$  or source entropy) or employing more receive antennas (higher implementation complexity), and the reduction rate of decoding thresholds slows down as  $N_r$  is incremented while the rate accelerates with entropy decreases. Therefore, sparse representation of sources plays a more important role than diversity orders in lowering decoding thresholds over Rayleigh fading channels, and the lower decoding thresholds suggest better error performance in the low-SNR region. Moreover, the capacity gaps between channel capacity and decoding thresholds for  $N_r = 1$  and 4 narrows as the source entropy goes down, i.e., the smallest gap (0.37 dB in the simulation) can be obtained for sparse sources with high diversity order.

Fig. 2 plots the bit error rate (BER) curves of DP\_LDPC codes over SIMO Rayleigh fading channel for different source statistics. It can be noted that the sparser source achieves better error performance for all diversity orders under study. More specifically, the DP\_LDPC scheme with one receive antenna

for source statistics  $P_1 = 0.01$  outperforms that with four receive antennas for source statistics  $P_1 = 0.02$  in the low-SNR region. Besides this, exploiting more receive antennas simply makes no sense to lower the error floor if source statistics being ignored. Moreover, the sparser source contributes more to improving convergence performance than the diversity order, due to less reduction on decoding thresholds can be obtained by increasing receive antennas. Accordingly, source statistics (sparsity) should be considered principally in order to design the appropriate capacity-approaching LDPC codes for JSCC systems over Rayleigh fading channels.

## V. CONCLUSION

In this letter, a joint PEXIT algorithm considering source statistics and multiple receive antennas for JSCC with protograph LDPC codes over Rayleigh fading channels has been developed. The decoding thresholds have also been analyzed by the presented joint PEXIT algorithm. Numerical results have revealed that source sparsity has a more important impact on decoding thresholds and BER performance than receive antenna diversity for JSCC systems in a fading situation.

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