A Serial Joint Channel and Physical Layer Network Decoding in Two-Way Relay Networks

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***Abstract*—This paper presents a novel joint channel and physical layer network decoding scheme for a two-way relay communication system. The proposed decoding scheme can be viewed as a serial concatenated decoding scheme with the source codes, different or identical, as the outer codes and the physical network coding (PNC) as the inner code. To achieve good error performance, iteratively decodable codes are used as the source codes. Decoding of the scheme consists of two loops, the inner-loop for which the source codes are decoded with a message-passing decoding algorithm and the outer-loop for which the reliability messages are passed between the source decoders and the PNC decoder for updating message reliabilities. Using two different protograph repeat-accumulate codes as the source codes for low decoder complexity, the proposed scheme achieves a superior performance. In the case that the source codes are identical, the proposed scheme outperforms the previously proposed decoding schemes for two-way relay communication systems with two source nodes.**

***Index Terms*—Physical network coding, relay, channel code.**

I. Introduction

IT WAS shown that physical network coding (PNC) can further increase the throughput of a network system over the conventional network coding [1]. Recently, several PNC schemes in conjunction with channel decoding have been proposed [2]–[4] for two-way relay communication systems with two source nodes. These schemes are referred as the joint channel decoding and physical network coding (JCNC) schemes which can be regarded as cross-layer designs for integrating the operations of the physical and network layers. These schemes were designed mainly for the case that the two sources are encoded with the same iteratively decodable code, such as a repeat-accumulate (RA) or an low-density parity-check (LDPC) codes. As pointed out in [5], [6], the key feature of such scheme is to employ the same code at two sources. Since the source codes are identical, the vector obtained from the XOR operations on the bits of the two codewords transmitted from the two sources is another codeword in the source code being used. Consequently, the same decoder can be used at each source node to decode the received vector to estimate the XOR information and the message transmitted from the other source. Using the same code and the same decoder for the source nodes, while simplifying the decoding complexity, put a constraint on the overall information transmission rate.

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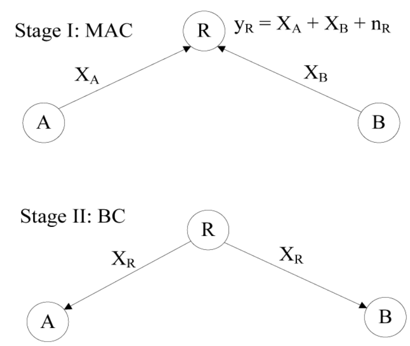

Fig. 1. Two sources A and B expect to exchange information from each other through the relay R, which comprises two stages: multiple access (MAC) and broadcast (BC).

Furthermore, the proposed schemes in [2]–[4] do not allow the relay node to perform additional operations to enhance the reliabilities of the individual messages.

In this paper, we presents a novel serial joint channel and physical network decoding (S-JCND) scheme for two-way relay communication systems for which the two source codes *can be either different or identical*. The S-JCND is constructed with the two source codes as outer codes and the PNC as the inner code. As the previously proposed JCNC schemes, iteratively decodable codes are used as the source codes. Decoding of the scheme consists of two iterative loops, the inner-loop for which the source codes are decoded with a message-passing decoding algorithm, say the sum-product algorithm (SPA), and the outer-loop for which the extrinsic information is passed between the source decoders and the PNC decoder for updating message reliabilities.

We show that using two different protograph RA codes [7] as the source codes, the proposed scheme achieves a superior performance. In the case for which the source codes are identical, the proposed scheme outperforms the previously proposed schemes [2]–[4] for two-way relay communication systems.

II. Background

The two-way relay network is a basic network structure of many attractions to wireless communication systems. As illustrated in Fig. 1, this system comprises two sources A, B and one relay R, where both sources expect to obtain information from each other through R.

The two-way relay communication includes two stages: multiple access (MAC) and broadcast (BC). In the MAC stage, let $\mathbf{b}_A$ and $\mathbf{b}_B$ denote the information vectors of length K of source A and B. The decodable channel codes are used to encode $\mathbf{b}_A$ and $\mathbf{b}_B$ into the codeword vectors $\mathbf{c}_A$ and $\mathbf{c}_B$ of

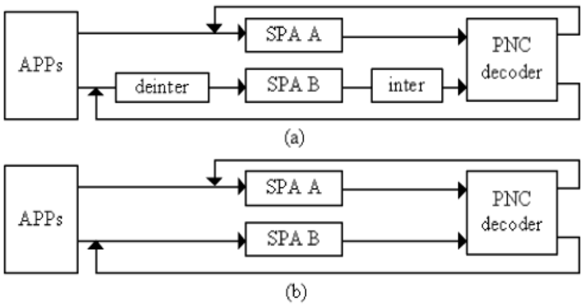

Fig. 2. The S-JCND decoding process in two cases, (a) the same source codes and (b) the different source codes, are employed at two sources.

length N with the same code rate. This encoding process is called the outer encoding. The $\mathbf{c}_{\mathrm{A}}$ and $\mathbf{c}_{\mathrm{B}}$ are BPSK modulated to vectors $\mathbf{x}_{\mathrm{A}}$ and $\mathbf{x}_{\mathrm{B}}$. Both sources A and B send $\mathbf{x}_{\mathrm{A}}$ and $\mathbf{x}_{\mathrm{B}}$ to the relay simultaneously over an AWGN channel. The output of the MAC channel is denoted by

$$\mathbf{y}_{\mathrm{R}} = \mathbf{x}_{\mathrm{A}} + \mathbf{x}_{\mathrm{B}} + \mathbf{n}_{\mathrm{R}} \quad (1)$$

where $\mathbf{n}_{\mathrm{R}}$ denotes the noise vector, whose elements are i.i.d zero-mean Gaussian random variables with variance σ^2. In the absence of noise, the channel output, i.e., the PNC, $\mathbf{x}_{\mathrm{A}} + \mathbf{x}_{\mathrm{B}}$ is called the inner encoding, which is similar to the coding for two-user multiple-access adder channel [8], [9]. By some certain decoding schemes, regarding the XOR information $\mathbf{b}_{\mathrm{A}\oplus\mathrm{B}} = \mathbf{b}_{\mathrm{A}} \oplus \mathbf{b}_{\mathrm{B}}$, the vector $\mathbf{b}_{\mathrm{R}} = \hat{\mathbf{b}}_{\mathrm{A}\oplus\mathrm{B}}$ is estimated at the relay. For instance, in the previous G-JCNC scheme [3], considering the same codes employed at the sources, a virtual encoder about this multi-access transmission is constructed with input $\mathbf{b}_{\mathrm{A}}+\mathbf{b}_{\mathrm{B}}$ and output $\mathbf{c}_{\mathrm{A}}+\mathbf{c}_{\mathrm{B}}$. Meanwhile, a decoding algorithm is correspondingly derived with the help of SPA. In this way, the $\mathbf{b}_{\mathrm{A}} + \mathbf{b}_{\mathrm{B}}$ is first decoded from the $\mathbf{y}_{\mathrm{R}}$, and then directly mapped to the desired $\mathbf{b}_{\mathrm{R}}$. That is, the individual information $\mathbf{b}_{\mathrm{A}}$ and $\mathbf{b}_{\mathrm{B}}$ are not necessary to be obtained separately. Then, the $\mathbf{b}_{\mathrm{R}}$ is encoded and BPSK-modulated into $\mathbf{x}_{\mathrm{R}}$, which is broadcasted by the relay in the BC stage. As the previous researches in PNC system, in this letter, we focus on the design of decoding algorithm to extract the $\mathbf{b}_{\mathrm{R}}$ in the first MAC stage.

III. The Proposed Serial Joint Decoding Involving PNC Decoding

Based on the above-described system model, we assume that the receiver at the relay knows both the source codes. Fig. 2 illustrates a serial concatenated decoding system, where the physical network coding is viewed as the inner code, and the two source codes are seen as the outer codes. The SPA A and SPA B denote the respective sum-product decoders for the codes used at the sources A and B. It should be noted that analogous to turbo codes [10], if the sources use the same code, one of the encoded codeword vectors will be interleaved before transmitted to the relay, so as to lower correlation between the two codewords when decoding, as shown in Fig. 2(a). The interleaver is not required if the sources use the different codes, as shown in Fig. 2(b).

Let J and L denote the iteration numbers of the inner loop and the outer loop. Given the received signal $\mathbf{y}_{\mathrm{R}}$, the detailed iterative decoding procedure of S-JCND scheme is performed in the following steps:

Step 1: *Message Initialization*. Similar to [2], in the absence of noise, it is assumed that the transmitted signal $\mathbf{x}_{\mathrm{A+B}}(n) = \mathbf{x}_{\mathrm{A}}(n)+\mathbf{x}_{\mathrm{B}}(n)$, $\{\mathbf{x}_{\mathrm{A+B}}(n) = 0, -2, 2\}$, the XOR of the source codewords $\mathbf{c}_{\mathrm{A}\oplus\mathrm{B}} = \mathbf{c}_{\mathrm{A}}(n)\oplus\mathbf{c}_{\mathrm{B}}(n)$, $\{\mathbf{c}_{\mathrm{A}\oplus\mathrm{B}} = 0, 1\}$, the sum of the codewords $\mathbf{c}_{\mathrm{A+B}}(n) = \mathbf{c}_{\mathrm{A}}(n)+\mathbf{c}_{\mathrm{B}}(n)$, $\{\mathbf{c}_{\mathrm{A+B}}(n) = 0, 1, 2\}$ with a-prior probabilities {1/2, 1/4, 1/4}, where n index bits of the vectors. For conciseness, let three vectors $\mathbf{p}_i$ correspond to probabilities $\Pr\{\mathbf{c}_{\mathrm{A+B}}(n) = i|\mathbf{y}_{\mathrm{R}}(n)\}$, $i = 0, 1, 2$. The probability of each element in $\mathbf{c}_{\mathrm{A+B}}$ can be calculated:

$$\mathbf{p}_0(n) = \frac{1}{4\sqrt{2\pi}\Pr\{\mathbf{y}_{\mathrm{R}}(n)\}}\exp(-\frac{(\mathbf{y}_{\mathrm{R}}(n)-2)^2}{2\sigma_n^2}) \quad (2)$$

$$\mathbf{p}_1(n) = \frac{1}{2\sqrt{2\pi}\Pr\{\mathbf{y}_{\mathrm{R}}(n)\}}\exp(-\frac{\mathbf{y}_{\mathrm{R}}(n)^2}{2\sigma_n^2}) \quad (3)$$

$$\mathbf{p}_2(n) = \frac{1}{4\sqrt{2\pi}\Pr\{\mathbf{y}_{\mathrm{R}}(n)\}}\exp(-\frac{(\mathbf{y}_{\mathrm{R}}(n)+2)^2}{2\sigma_n^2}) \quad (4)$$

where $\Pr\{\mathbf{y}_{\mathrm{R}}(n)\}$ is the probability that $\mathbf{y}_{\mathrm{R}}(n)$ is transmitted, which can be seen as a normalized factor to satisfy that the sum of the probabilities in (2)−(4) equals one. Let subscript M mark the two sources, M = A, B. Because of the symmetry of the source codewords, the a-posteriori probabilities (APPs) soft information $\Pr\{\mathbf{c}_{\mathrm{M}}(n)|\mathbf{y}_{\mathrm{R}}(n)\}$ of the two decoders are the same and calculated by:

$$\Pr\{\mathbf{c}_{\mathrm{M}}(n) = 0|\mathbf{y}_{\mathrm{R}}(n)\} = \beta \times (\mathbf{p}_0(n) + 0.5 \times \mathbf{p}_1(n)) \quad (5)$$

$$\Pr\{\mathbf{c}_{\mathrm{M}}(n) = 1|\mathbf{y}_{\mathrm{R}}(n)\} = \beta \times (\mathbf{p}_2(n) + 0.5 \times \mathbf{p}_1(n)) \quad (6)$$

where β is a normalized factor such that the sum of two probabilities could be 1. The APPs are then fed into the respective decoders, and as shown in Fig. 2(a), for the case with the same source codes, the resulting APPs for SPA B should be de-interleaved in advance.

Step 2: *Outer Channel Code Decoding*. Using inputs of the APPs and *the extrinsic information* from PNC decoding, the SPA M is executed with its own parity matrix by J iteration numbers, M = A, B. Let $\mathbf{q}_{\mathrm{M}}^i$ denote the *extrinsic information* vector of the codeword with value i from SPA M, $i = 0, 1$. If all parity check equations of both decoders are satisfied, the algorithm stops, otherwise, the $\mathbf{q}_{\mathrm{M}}^i$ are passed into PNC decoder. For the case with the same source codes, the $\mathbf{q}_{\mathrm{B}}^i$ of SPA B enter the PNC decoder after interleaving.

Step 3: *Inner PNC Decoding*. Given the input extrinsic information $\mathbf{q}_{\mathrm{M}}^i$ and the channel information, i.e., $\mathbf{p}_0$, $\mathbf{p}_1$ and $\mathbf{p}_2$, the message reliabilities of the two codewords are updated in this step according to the essence of PNC coding: $\mathbf{c}_{\mathrm{A+B}}(n) = \mathbf{c}_{\mathrm{A}}(n) + \mathbf{c}_{\mathrm{B}}(n)$. Let $\mathbf{w}_{\mathrm{M}}^i$ denote the *extrinsic information* vector for the codeword of value i in SPA M, $i = 0, 1$, M = A, B. The $\mathbf{w}_{\mathrm{M}}^i$ are computed in (7)-(10), where β_{A} and β_{B} are used for probability normalization. The algorithm stops if the maximum outer iteration L is reached. Otherwise, it proceeds to Step 2 with the $\mathbf{w}_{\mathrm{A}}^i$ for SPA A and the $\mathbf{w}_{\mathrm{B}}^i$ for SPA B. Specifically, the $\mathbf{w}_{\mathrm{B}}^i$ is de-interleaved before entering SPA B in the case with the identical source codes.

Step 4: *Decision.* When iteration stops, let $\hat{\mathbf{b}}_{\mathrm{M}}$ denote the information decision vector of SPA M and k index a bit of

$$\begin{aligned}\mathbf{w}_{\mathrm{A}}^{0}(n) &= \beta_{\mathrm{A}} \times (\Pr\{\mathbf{c}_{\mathrm{B}}(n)=0, \mathbf{c}_{\mathrm{A+B}}(n)=0|\mathbf{q}_{\mathrm{B}}^{0}(n), \mathbf{y}_{\mathrm{R}}(n)\} + \Pr\{\mathbf{c}_{\mathrm{B}}(n)=1, \mathbf{c}_{\mathrm{A+B}}(n)=1|\mathbf{q}_{\mathrm{B}}^{1}(n), \mathbf{y}_{\mathrm{R}}(n)\}) \\ &= \beta_{\mathrm{A}} \times (\mathbf{q}_{\mathrm{B}}^{0}(n) \times \mathbf{p}_0(n) + 0.5 \times \mathbf{q}_{\mathrm{B}}^{1}(n) \times \mathbf{p}_1(n)) \end{aligned} \tag{7}$$

$$\begin{aligned}\mathbf{w}_{\mathrm{A}}^{1}(n) &= \beta_{\mathrm{A}} \times (\Pr\{\mathbf{c}_{\mathrm{B}}(n)=1, \mathbf{c}_{\mathrm{A+B}}(n)=2|\mathbf{q}_{\mathrm{B}}^{1}(n), \mathbf{y}_{\mathrm{R}}(n)\} + \Pr\{\mathbf{c}_{\mathrm{B}}(n)=0, \mathbf{c}_{\mathrm{A+B}}(n)=1|\mathbf{q}_{\mathrm{B}}^{0}(n), \mathbf{y}_{\mathrm{R}}(n)\}) \\ &= \beta_{\mathrm{A}} \times (\mathbf{q}_{\mathrm{B}}^{1}(n) \times \mathbf{p}_2(n) + 0.5 \times \mathbf{q}_{\mathrm{B}}^{0}(n) \times \mathbf{p}_1(n)) \end{aligned} \tag{8}$$

$$\begin{aligned}\mathbf{w}_{\mathrm{B}}^{0}(n) &= \beta_{\mathrm{B}} \times (\Pr\{\mathbf{c}_{\mathrm{A}}(n)=0, \mathbf{c}_{\mathrm{A+B}}(n)=0|\mathbf{q}_{\mathrm{A}}^{0}(n), \mathbf{y}_{\mathrm{R}}(n)\} + \Pr\{\mathbf{c}_{\mathrm{A}}(n)=1, \mathbf{c}_{\mathrm{A+B}}(n)=1|\mathbf{q}_{\mathrm{A}}^{1}(n), \mathbf{y}_{\mathrm{R}}(n)\}) \\ &= \beta_{\mathrm{B}} \times (\mathbf{q}_{\mathrm{A}}^{0}(n) \times \mathbf{p}_0(n) + 0.5 \times \mathbf{q}_{\mathrm{A}}^{1}(n) \times \mathbf{p}_1(n)) \end{aligned} \tag{9}$$

$$\begin{aligned}\mathbf{w}_{\mathrm{B}}^{1}(n) &= \beta_{\mathrm{B}} \times (\Pr\{\mathbf{c}_{\mathrm{A}}(n)=1, \mathbf{c}_{\mathrm{A+B}}(n)=2|\mathbf{q}_{\mathrm{A}}^{1}(n), \mathbf{y}_{\mathrm{R}}(n)\} + \Pr\{\mathbf{c}_{\mathrm{A}}(n)=0, \mathbf{c}_{\mathrm{A+B}}(n)=1|\mathbf{q}_{\mathrm{A}}^{0}(n), \mathbf{y}_{\mathrm{R}}(n)\}) \\ &= \beta_{\mathrm{B}} \times (\mathbf{q}_{\mathrm{A}}^{1}(n) \times \mathbf{p}_2(n) + 0.5 \times \mathbf{q}_{\mathrm{A}}^{0}(n) \times \mathbf{p}_1(n)) \end{aligned} \tag{10}$$

$\hat{\mathbf{b}}_{\mathrm{M}}$. The $\hat{\mathbf{b}}_{\mathrm{M}}(k)$ is estimated by

$$\hat{\mathbf{b}}_{\mathrm{M}}(k) = \begin{cases} 1 & \text{if } \Pr\{\hat{\mathbf{b}}_{\mathrm{M}}(k)=1\} \geq 0.5 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Finally, the network-coded information can be obtained, i.e., $\mathbf{b}_{\mathrm{R}}(k) = \hat{\mathbf{b}}_{\mathrm{A}}(k) \oplus \hat{\mathbf{b}}_{\mathrm{B}}(k)$.

Note that the S-JCND scheme adopts two decoders and an interleaver to carry out iterative decoding. It has the increased hardware complexity as compared to the G-JCNC scheme because the latter only uses one decoder to acquire the desired information.

IV. Examples and Simulations

In this section, the performance of the proposed scheme is investigated through several simulations. We firstly consider the case where the two sources use different protograph systematic RA codes. With the code rate-0.25 and the repeat factor of value 3, the codes are respectively constructed for block sizes (N, K) = (2048, 512) and (N, K) = (4096, 1024). A simple separate channel decoding scheme (SCD), which was mentioned in [4] for such a case is also simulated for comparison. With total 120 decoding iterations, the S-JCND and SCD are performed to obtain bit error rate (BER) of the decoded block $\mathbf{b}_{\mathrm{R}}$. Fig. 3 shows that the proposed scheme performs well while the SCD performs badly. Furthermore, the values of J and K have an affect on the performance of the S-JCND. The best performance is achieved when (J, L) is set to (30, 4). In particular, we also consider the system for which the sources use the same source RA code. Assuming the uniform random interleaver (URI) is applied, Fig. 4 depicts the performance of G-JCNC and S-JCND with (J, L) of (30, 4). It is notable that the S-JCND show a significant gain about 0.6 dB over the G-JCNC counterparts for both K = 512 and K = 1024.

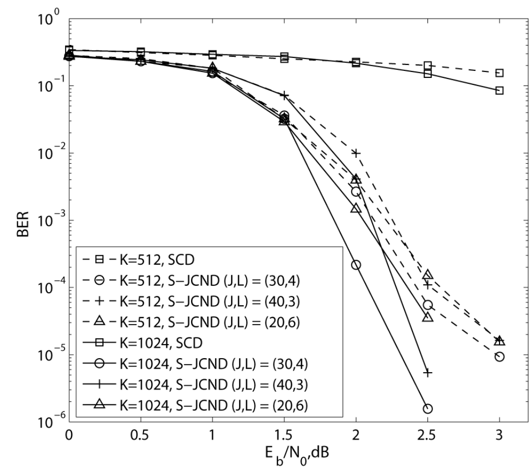

Fig. 3. The BER performance comparison between SCD and S-JCND with the different RA codes. Information lengths are 512 and 1024. Code rates are 0.25.

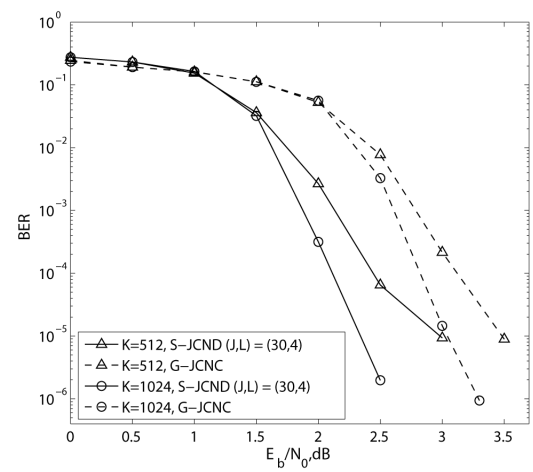

Fig. 4. The BER performance comparison between G-JCNC and S-JCND with the same RA codes. Information lengths are 512 and 1024. Code rates are 0.25.

To give more insights about the interleavers in the case with the same RA source codes, we simulate the S-JCND with different interleavers including s-random interleaver (SRI), where the spread s = 16, 32 and 45 for N = 512, 2048 and 4096 [11], and quasi-cyclic interleaver (QCI) [12]. The two interleavers were designed to improve performance of turbo codes. With iteration numbers (J, L) of (30, 4) and code lengths of 512, 2048 and 4096, their BER performance are compared in Fig. 5. One can observe that the system with the QCI slightly outperforms those with the URI or the SRI at high E_b/N_0 region. Moreover, it obtains a gain of half an order of magnitude over the one with the URI at $E_b/N_0 = 2.7$, when K equals 1024, i.e., the interleaver length N is 4096. In addition, the URI and the SRI are mostly random with weak determinism. Hence the QCI of certain determinism is the best for the proposed scheme in terms of performance and practicality.

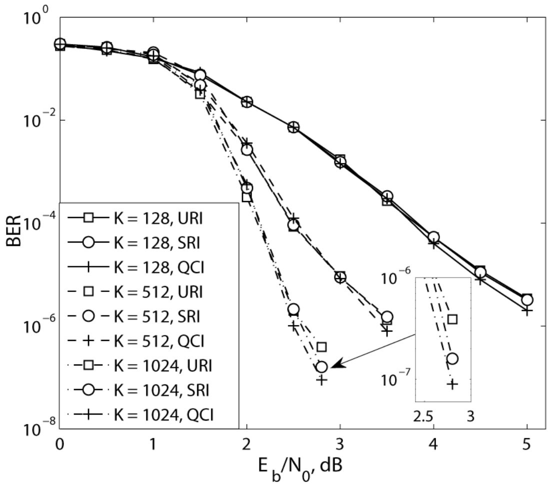

Fig. 5. The BER performance of S-JCND with different interleavers: URI, SRI and QCI, (J, L) = (30, 4). Information lengths are 128, 512 and 1024. Code rates are 0.25.

V. Conclusion

A serial joint channel and physical network decoding scheme was presented. Taking PNC decoding into account, it enables more generalized wireless communications over two-way relay channel since it allows the sources to use the same or different channel codes. Instead of the previous JCNC scheme directly estimating the XOR information, this proposed scheme recovers not only the desired XOR information, but also the individual source messages. Simulation results showed that the S-JCND exhibits excellent error performance if the source codes are different, and obtains substantial BER improvements over the G-JCNC if the source codes are identical. Meanwhile, the results indicated that the S-JNCD of the QCI is superior to the ones of the URI or the SRI with respect to performance and practicality.

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