

A NEW DATA RATE ADAPTION COMMUNICATIONS SCHEME FOR CODE-SHIFTED DIFFERENTIAL CHAOS SHIFT KEYING MODULATION*

W. K. XU and L. WANG[†]

*Shenzhen Research Institute of Xiamen University
Shenzhen, Guangdong Province 518057, China
xweikai@xmu.edu.cn wanglin@xmu.edu.cn[‡]*

G. KOLUMBÁN

*The Faculty of Information Technology, Pazmany Peter Catholic University,
Budapest, Hungary
kolumbán@ijtk.ppke.hu*

Received (to be inserted by publisher)

In a binary Transmitted Reference (TR) system each bit is encoded into two wavelets of finite duration. The information is carried by the sign of correlation measured between the two wavelets. The Code-Shifted Differential Chaos Shift Keying (CS-DCSK) modulation scheme transmits the two wavelets in the same time slot and applies two Walsh code sequences to keep the wavelets separated. The CS-DCSK modulation scheme is generalized here by transmitting more than one information bearing wavelets with one reference. The orthogonality of wavelets is assured by different Walsh code sequences. The new Generalized CS-DCSK (GCS-DCSK) scheme is a multilevel modulation where the symbol period is kept constant but the data rate can be varied in an adaptive manner by adding new or removing existing information bearing wavelets, each of them is isolated by Walsh code. Exploiting the Gaussian approximation, an analytical expression is derived for the noise performance of GCS-DCSK modulation. Its accuracy is verified by computer simulation.

Keywords: Chaos communications; Generalized code-shifted differential chaos shift keying; Rate adaption

1. Introduction

Chaos based communications theory and, particularly, chaotic modulation schemes have been a hot research topic recently where, contrary to conventional modulation schemes, a wideband, non-periodic chaotic signal is used as carrier [Lau & Tse, 2003]. The wideband power spectral density, the excellent auto-correlation and cross-correlation properties of chaotic wavelets equip the chaos-based modulation scheme with a high robustness against multipath fading even in severe multipath environment. To satisfy the demands of

*For the title, try not to use more than three lines. Typeset the title in 15 pt Times Roman, uppercase and boldface.

[†]Typeset names in 11 pt Times Roman. Use the footnote to indicate the present or permanent address of the author.

[‡]State completely without abbreviations the affiliation and mailing address, including country. Typeset in 11 pt Times Italic.

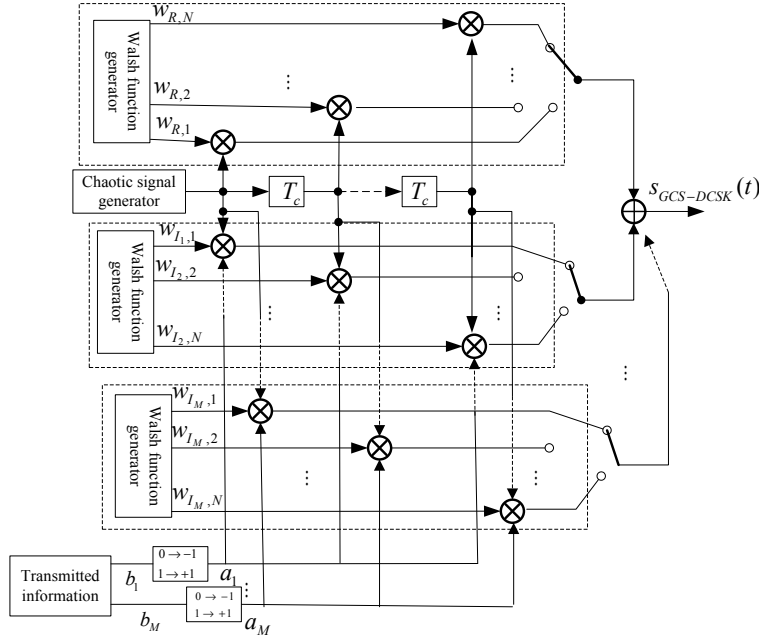


Fig. 1. Block diagram of the GCS-DCSK transmitter.

different applications, many chaotic modulation schemes have been proposed, analyzed and optimized [Dedieu *et al.*, 1993; Kolumban *et al.*, 1996, 1998; Kennedy *et al.*, 2000; Xia *et al.*, 2004; Ye *et al.*, 2005; Yao & Lawrance, 2006; Wang *et al.*, 2008], among which frequency-modulated differential chaos shift keying (FM-DCSK) modulation [Kolumban *et al.*, 1998] offers the best multipath performance. Binary DCSK and FM-DCSK belong to the class of transmitted-reference (TR) system [Rushforth, 1964] and offer an alternative solution to spread-spectrum (SS) communications [Ye *et al.*, 2005]. Binary DCSK and FM-DCSK can be demodulated with a simple autocorrelation receiver (AcR) that can capture the entire signal energy without requiring the acquisition and synchronization of a spreading code and channel estimation [Kolumban, 2000].

Exploiting wideband property of chaotic signal, ultra-wideband (UWB) communications systems implementing binary DCSK and FM-DCSK modulations have been proposed recently [Chong & Yong, 2008; Kolumban, 2005; Min *et al.*, 2010]. Unfortunately, DCSK and FM-DCSK receivers require Radio Frequency (RF) delay lines that are very difficult to implement because of the extremely wide bandwidth. To overcome this difficulty, code-shifted DCSK has been proposed in [Xu *et al.*, 2011] where the reference and information bearing wavelets of finite duration are transmitted in the same time slot and are separated by two Walsh code sequences. Consequently, there is no need for delay line at the CS-DCSK receiver.

In this paper, the CS-DCSK concept is extended to transmit multiple bit streams by means of one reference wavelet. In the generalized code-shifted DCSK (GCS-DCSK) modulation scheme introduced in Sec. 2 the reference wavelet and more than one information-bearing wavelets are transmitted in the same time slot. The symbol duration is kept constant and the wavelets are separated by Walsh code sequences.

The GCS-DCSK modulation scheme belongs to the class of multilevel modulations and, as shown in Sec. 2, its data rate may be varied by adding new or removing already existing information bearing wavelets. Hence, the GCS-DCSK modulation belongs to the class of rate-adaption schemes. Sec. 3 develops an analytical expression for the noise performance of GCS-DCSK over an Additive White Gaussian Noise (AWGN) channel. The analytical expression is verified in Sec. 4 by computer simulations.

2. SYSTEM MODEL

The remarkable advantage of CS-DCSK over DCSK is that the implementation of a CS-DCSK receiver does not require an RF delay line [Xu *et al.*, 2011]. The CS-DCSK scheme implements a binary modulation

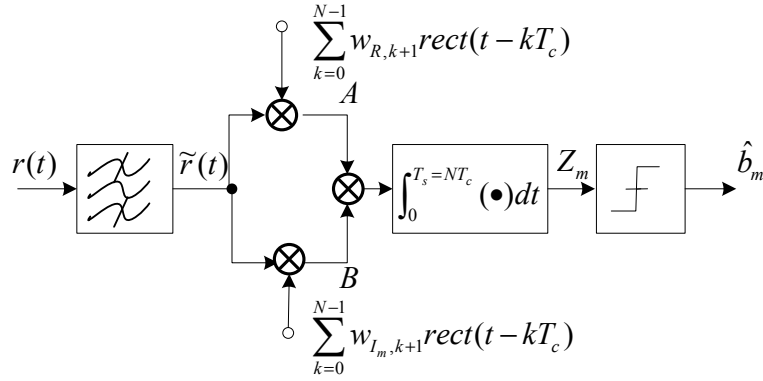


Fig. 2. Block diagram of m^{th} bit stream detector in a GCS-DCSK receiver.

since only one reference wavelet and one information-bearing wavelet are transmitted in each bit period.

The CS-DCSK concept can be generalized if the transmission of more than one information bearing wavelet is performed in one symbol period. The new modulation scheme, referred to as Generalized Code-Shifted DCSK (GCS-DCSK), is a multilevel modulation scheme that is suitable for parallel transmission of multiple bit streams. Only one reference waveform is used in each symbol period and the reference and information bearing waveforms are separated by the Walsh code sequences.

Consider the transmission of a single isolated symbol. Then the GCS-DCSK signal takes the form

$$s_{GCS-DCSK}(t) = \sum_{k=0}^{N-1} w_{R,k+1} c(t - kT_c) + \sum_{i=1}^M a_i \sum_{k=0}^{N-1} w_{I_i,k+1} c(t - kT_c), \quad T_s = NT_c \quad (1)$$

where the bits of the i^{th} stream ($i = 1, 2, \dots, M$) are mapped into $a_i \in \{-1, +1\}$, w_R and w_{I_i} , respectively, denote the Walsh code sequences used to identify the reference and i^{th} information bearing wavelets. The block diagram of GCS-DCSK transmitter derived from Eq. (1) is shown in Fig. 1.

Each GCS-DCSK receiver contains M bit stream detectors to recover the transmitted information. The m^{th} bit stream detector has same structure as CS-DCSK detector. Its block diagram is shown in Fig. 2.

The observation signal of the m^{th} bit stream detector is obtained as

$$Z_m = \int_0^{T_s=NT_c} \sum_{k=0}^{N-1} [\tilde{r}(t - kT_c) w_{R,k+1} \text{rect}(t - kT_c)] \times [\tilde{r}(t - kT_c) w_{I_m,k+1} \text{rect}(t - kT_c)] dt \quad (2)$$

Consider a noise- and distortion-free channel where

$$\tilde{r} = s_{GCS-DCSK}(t)$$

Substituting Eq. (1) into Eq. (2) and, exploiting the orthogonality of Walsh code sequences one obtain

$$Z_m = \int_0^{T_s} \sum_{k=0}^{N-1} c(t - kT_c) a_m c(t - kT_c) dt = a_m \int_0^{T_s} \sum_{k=0}^{N-1} c^2(t - kT_c) dt \quad (3)$$

where a_m carries the digital information transmitted and the integral on the RHS, positive number, gives the energy of chaotic wavelet.

In each bit stream the transmitted bit is recovered by a decision circuit

$$\hat{b}_m = \begin{cases} 1, & \text{for } Z_m \geq 0 \\ 0, & \text{for } Z_m < 0 \end{cases} \quad (4)$$

Remarks:

In many applications the data communications has to be maintained all the time and the worsening channel conditions can be compensated by reducing the data rate. To satisfy this demand simple modulation schemes and transceiver configurations are required in mobile communications that are equipped with the rate-adaption capability [Andrews *et al.*, 2007].

The GCS-DCSK modulation scheme offers this feature. To get the simplest transceiver configuration, both the chip time T_c and symbol duration T_s are fixed in a built GCS-DCSK system. Because M information bearing wavelets are transmitted in one symbol period, Eq. (1) defines a multilevel modulation scheme where the data rate is M times higher than that of the CS-DCSK scheme. The orthogonality of wavelets is assured by the Walsh code sequences, consequently, the number of information bearing wavelets can be varied at any time even in a running system without affecting the data transmission. This is why the GCS-DCSK modulation scheme is suitable for the implementation of a rate-adaption data communications system.

3. PERFORMANCE ANALYSIS

To evaluate the noise performance of the GCS-DCSK modulation scheme let an analytical expression be developed for its noise performance. Assume that the received signal $r(t)$ is corrupted by an Additive White Gaussian (AWG) noise in the channel and assume that the mean and variance of channel noise are zero and $N_0/2$, respectively.

Let $n(t)$ denote the sample function of channel noise. Since the channel filter does not distort the GCS-DCSK signal, the signal at the output of channel filter takes the form

$$\tilde{r}(t) = s_{GCS-DCSK}(t) + \tilde{n}(t) \quad (5)$$

Substituting Eq. (5) into Eq. (2) and exchanging the order of integration and summation, the observation signal for an AWGN channel is obtained as

$$\begin{aligned} Z_m &= \sum_{k=0}^{N-1} \int_{kT_c}^{(k+1)T_c} w_{R,k+1} \tilde{r}(t) w_{I_m,k+1} \tilde{r}(t) dt \\ &= \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} \int_{kT_c}^{(k+1)T_c} \tilde{r}^2(t) dt \\ &= \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} \\ &\quad \times \int_{kT_c}^{(k+1)T_c} \left[\left(w_{R,k+1} + \sum_{i=1}^M a_i w_{I_i,k+1} \right) c(t - kT_c) \right. \\ &\quad \left. + \tilde{n}(t) \right]^2 dt \end{aligned} \quad (6)$$

Because the chaotic wavelet $c(t)$ is generated here by a difference equation, a discrete-time equivalent of GCS-DCSK system with a sampling rate of f_s is used in the remaining part of performance analysis. In order to clearly distinguish the analog GCS-DCSK system from its discrete-time equivalent, the discrete-time chaotic wavelet and samples of channel noise are denoted by x_j and η_j , respectively. The relationship between the analog signals of finite duration and their samples are given by

$$x_j = c([j - 1]t_s) \quad \text{and} \quad \eta_j = n([j - 1]t_s)$$

where $j = 1, 2, \dots, \beta$ and $t_s = 1/f_s$ is the sample interval.

The discrete-time chaotic wavelets are generated by a Logistic map

$$x_{j+1} = 1 - 2x_j^2 \quad (7)$$

The number of samples generated in one chip period is $\beta = T_c f_s$. The spreading factor (SF = $T_s f_s$) gives the number of samples that are accommodated in one symbol period.

Using the notation discussed above, the discrete-time equivalent of Eq. (6) is obtained as

$$Z_m = \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} \times \sum_{j=1}^{\beta} \left[\left(w_{R,k+1} + \sum_{i=1}^M a_i w_{I_i,k+1} \right) x_{k\beta+j} + \eta_{k\beta+j} \right]^2 \quad (8)$$

The observation signal can be decomposed into three terms

$$Z_m = Z_{s \times s} + Z_{s \times n} + Z_{n \times n} \quad (9)$$

$$\begin{aligned} Z_{s \times s} &= \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} \sum_{j=1}^{\beta} \left[(w_{R,k+1} + a_1 w_{I_1,k+1} + \dots + a_m w_{I_m,k+1} + \dots + a_M w_{I_M,k+1}) \right]^2 x_{k\beta+j}^2 \\ &= 2 \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} a_m w_{R,k+1} w_{I_m,k+1} \sum_{j=1}^{\beta} x_{k\beta+j}^2 \\ &\quad + 2 \sum_{i=1}^{M-1} \sum_{l=i+1}^M a_i a_l \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} w_{I_i,k+1} w_{I_l,k+1} \sum_{j=1}^{\beta} x_{k\beta+j}^2 \\ &\quad + \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} w_{R,k+1}^2 \sum_{j=1}^{\beta} x_{k\beta+j}^2 + \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} \sum_{i=1}^M a_i^2 w_{I_i,k+1}^2 \sum_{j=1}^{\beta} x_{k\beta+j}^2 \\ &\quad + 2 \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} \sum_{i=1, i \neq m}^M a_i w_{R,k+1} w_{I_i,k+1} \sum_{j=1}^{\beta} x_{k\beta+j}^2 \\ &= 2N a_m \sum_{j=1}^{\beta} x_j^2 + \underbrace{2 \sum_{i=1}^{M-1} \sum_{l=i+1}^M a_i a_l \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} w_{I_i,k+1} w_{I_l,k+1} \sum_{j=1}^{\beta} x_{k\beta+j}^2}_{=MSI} \\ &\quad + \underbrace{\sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} \sum_{j=1}^{\beta} x_j^2}_{=0} + \underbrace{\sum_{i=1}^M \sum_{k=0}^{N-1} w_{R,k+1} w_{I_i,k+1} \sum_{j=1}^{\beta} x_j^2}_{=0} + \underbrace{2 \sum_{i=1, i \neq m}^M \sum_{k=0}^{N-1} a_i w_{R,k+1}^2 w_{I_i,k+1} w_{I_m,k+1} \sum_{j=1}^{\beta} x_j^2}_{=0} \end{aligned} \quad (10)$$

$$\begin{aligned}
Z_{s \times n} &= 2 \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} \sum_{j=1}^{\beta} (w_{R,k+1} + a_1 w_{I_1,k+1} + a_2 w_{I_2,k+1} + \dots + a_m w_{I_m,k+1} + \dots + a_M w_{I_M,k+1}) x_{k\beta+j} \eta_{k\beta+j} \\
&= 2 \sum_{k=0}^{N-1} w_{R,k+1}^2 w_{I_m,k+1} \sum_{j=1}^{\beta} x_{k\beta+j} \eta_{k\beta+j} + 2 \sum_{i=1, i \neq m}^M a_i \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} w_{I_i,k+1} \times \sum_{j=1}^{\beta} x_{k\beta+j} \eta_{k\beta+j} \\
&\quad + 2a_m \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1}^2 \sum_{j=1}^{\beta} x_{k\beta+j} \eta_{k\beta+j} \\
&= 2 \sum_{k=0}^{N-1} w_{I_m,k+1} \sum_{j=1}^{\beta} x_j \eta_{k\beta+j} + 2 \sum_{i=1, i \neq m}^M a_i \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} w_{I_i,k+1} \sum_{j=1}^{\beta} x_j \eta_{k\beta+j} + 2a_m \sum_{k=0}^{N-1} w_{R,k+1} \sum_{j=1}^{\beta} x_j \eta_{k\beta+j}
\end{aligned} \tag{11}$$

$$Z_{n \times n} = \sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} \sum_{j=1}^{\beta} (\eta_{k\beta+j})^2 \tag{12}$$

Variables $Z_{s \times s}$, $Z_{s \times n}$ and $Z_{n \times n}$ defined by Eq. (10), Eq. (11) and Eq. (12), respectively, are random variables where $Z_{s \times s}$ depends on the transmitted GCS-DCSK signal, $Z_{s \times n}$ gives the cross-correlation between the transmitted signal and channel noise, $Z_{n \times n}$ depends exclusively on channel noise.

RHS of Eq. (10) contains the useful term that carries the modulation to be recovered and a term expressing the interference among the different bit streams. This term is referred to as Multiple Streams Interference (MSI). Note, MSI vanishes if the code sequences used to separate the wavelets meet the following condition

$$\sum_{k=0}^{N-1} w_{R,k+1} w_{I_m,k+1} w_{I_i,k+1} w_{I_l,k+1} = 0 \tag{13}$$

Recall, in the above equation i and m identify the transmitted bit stream [see Eq. (1)] and the bit stream detector [see Eq. (2)], respectively, and $l = i + 1$. where $i, l, m = 1, 2, \dots, M$ are the subscript of stream. Other terms are zeros due to the orthogonality of Walsh code sequences.

In GCS-DCSK proposed here the Walsh code sequences constructed by a Hadamard matrix are used. Condition Eq. (13) is met if the length N of a Walsh code sequence is a power of 2 and the code sequences appearing in Eq.(13) are chosen from the rows of an $N \times N$ -size Hadamard matrix. If the reference sequence w_R is chosen from the first $N/2$ rows, then the M sequences assigned for the information bearing wavelets can be taken arbitrarily from the last $N/2$ rows. As a total, $M + 1$ sequences are required, consequently, MSI can be eliminated if and only if $M \leq N/2$.

Assume that a bit "1" is transmitted. Random variables $Z_{s \times s}$, $Z_{s \times n}$ and $Z_{n \times n}$ are independent from one another. Consequently, the mean and variance of observation signal is obtained as

$$E\{Z_m | a_m = +1\} = 2N\beta E\{x_j^2\} \tag{14}$$

$$\begin{aligned}
\text{var}\{Z_m | a_m = +1\} &= 2N\beta \text{var}\{x_j^2\} \\
&\quad + (M + 1)2NN_0\beta E\{x_j^2\} + N\beta N_0^2
\end{aligned} \tag{15}$$

where $E\{\cdot\}$ and $\text{var}\{\cdot\}$ represent the expectation and variance operators, respectively.

When a bit "0" is transmitted then the mean and variance of observation signal is derived in a similar manner

$$E\{Z_m | a_m = -1\} = -E\{Z_m | a_m = +1\} \tag{16}$$

$$\text{var}\{Z_m | a_m = -1\} = \text{var}\{Z_m | a_m = +1\} \quad (17)$$

Observe, the mean and variance of observation variable depends on the parameters of chaotic wavelets. For Logistic map we get $E\{x_j^2\} = 0.5$ and $\text{var}\{x_j^2\} = 0.125$ [Lau & Tse, 2003]. Then the BER can be expressed using the Gaussian (GA) approximation

$$\begin{aligned} BER &= \frac{1}{2}\Pr(Z_m < 0 | a_m = +1) + \frac{1}{2}\Pr(Z_m \geq 0 | a_m = -1) \\ &= \frac{1}{2}\text{erfc}\left(\frac{E\{Z_m | a_m = +1\}}{\sqrt{2\text{var}\{Z_m | a_m = +1\}}}\right) \\ &= \frac{1}{2}\text{erfc}\left(\frac{2N\beta E\{x_j^2\}}{\sqrt{2\left(2N\beta\text{var}\{x_j^2\} + 2(M+1)NN_0\beta E\{x_j^2\} + N\beta N_0^2\right)}}\right) \\ &= \frac{1}{2}\text{erfc}\left(\left[2\left(\frac{\text{var}\{x_j^2\}}{2N\beta E^2\{x_j^2\}} + \frac{(M+1)^2 N_0}{2(M+1)N\beta E^2\{x_j^2\}}\right.\right.\right. \\ &\quad \left.\left.\left. + \frac{(M+1)^2 N\beta N_0^2}{4(M+1)^2 N^2 \beta^2 E^2\{x_j^2\}}\right)\right]^{-1/2}\right) \\ &= \frac{1}{2}\text{erfc}\left(\left[\frac{1}{2N\beta} + \frac{(M+1)^2}{E_s/N_0} + \frac{(M+1)^2 N\beta}{2(E_s/N_0)^2}\right]\right) \end{aligned} \quad (18)$$

In Eq. (18), $E_s = (M+1)N\beta E\{x_j^2\} = ME_b$ denotes the average energy per symbol. Unfortunately, due to the chaotic carrier E_s is not constant, it varies even if the same symbol is transmitted repeatedly. Note, the variance in E_s increases the variance of observation signal and deteriorates the noise performance of the GCS-DCSK system. This problem can be prevented by keeping the energy of each chaotic waveform constant [Kolumbán *et al.*, 1998].

4. SIMULATIONS

BER performances of GCS-DCSK modulation are evaluated through Monte-Carlo simulation and numerical calculation in this Section. During the simulations, the 3rd order Walsh codes are used to assure orthogonality of transmitted wavelets and the condition of Eq. (13) is always satisfied in order to cancel MSI. Ideal synchronization is assumed, that is the position of the observation window is experimentally set at the receiver to minimize the BER for each E_b/N_0 value.

Figure 3-4 compared the theoretical BER curves and their simulated counterparts for the spreading factors 80 and 24, respectively. Here, the 3rd order Walsh code sequence ($N = 8$) is used, and the numbers of transmitted bit streams are 2,3 and 4, respectively. The theoretical and simulated results are in a very good agreement when the spreading factor is large enough. However, because of the non periodic nature of chaotic signals, the transmitted bit energy the proposed system varies from one bit to another. Gaussian approximation is feasible under the assumption of the transmitted chaotic bit energy being constant. This can be shown in Fig. 4 when the spreading factor is not very large. The exact BER computation can be obtained using semi-analytical method [Kaddoum *et al.*, 2009], when the spread factor is small. Note, an increase in the number of bit streams results in a negligible degradation in BER performance but in an increase in the data rate. Thus, a trade-off exists between the data rate and noise performance.

5. Conclusion

The Generalized Code-Shifted Differential Chaos Shift Keying modulation scheme has been derived here. GCS-DCSK belongs to the class of transmitted reference systems where the information is mapped into

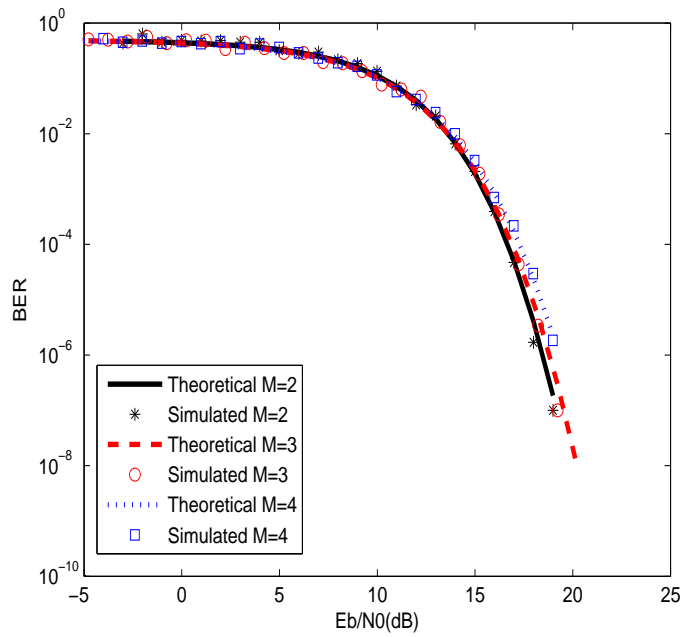


Fig. 3. BER vs. E_b/N_0 performance curves of the 2-, 3- and 4-stream GCS-DCSK systems over AWGN channel when a 3rd order Walsh code sequence is used to separate the transmitted wavelets. The spreading factor was set to 80.

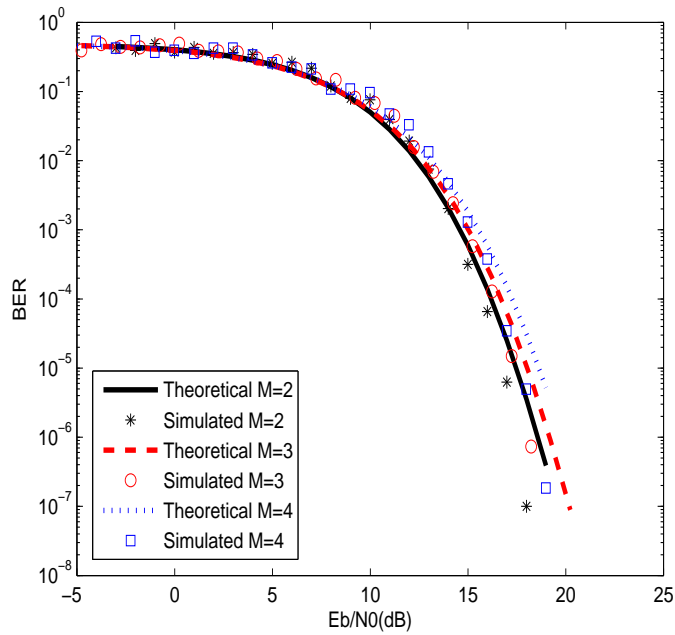


Fig. 4. BER vs. E_b/N_0 performance curves of the 2-, 3- and 4-stream GCS-DCSK modulation schemes over AWGN channel when a 3rd order Walsh code is used to separate the transmitted wavelets. The spreading factor was set to 24.

two wavelets, a reference wavelet and an information bearing one. The information is recovered from the sign of correlation measured between the reference and information bearing wavelets.

GCS-DCSK has three novelties, (i) more than one information bearing wavelets are transmitted with one reference wavelet, (ii) the reference and information bearing wavelets are transmitted in the same time

slot and (iii) the reference and information bearing wavelets are separated by the Walsh code sequences.

GCS-DCSK is a multilevel modulation scheme that has a unique property: because of the orthogonality of Walsh code sequences the number of information bearing wavelets may be changed during data transmission without causing any interruption. Consequently, GCS-DCSK implements a rate-adaption modulation scheme.

Acknowledgment

This work was supported by National Natural Science Foundation of China (No. 61001073 and No. 60972053), the R&D Funding for Basic Research Program of Shenzhen (No. JC201006030862A) and the Project of (CSTC) Chongqing Science & Technology Commission (No. 2010AC3060). G. Kolumbán's participation in the research was financed by the Hungarian Scientific Research Fund (OTKA) under Grant number K-84045.

References

- Xu W. K., Wang L. & Kolumbán G. [2011] "A novel Differential Chaos Shift Keying Modulation Scheme," *Int. J. Bifurcation and Chaos* **21**, 799–814.
- Lau, F. C. M. & Tse, C. K [2003] *Chaos-based Digital Communication Systems: Operating Principles, Analysis Methods, and Performance Evaluation*, 1nd Ed. (Springer-Verlag Press, Berlin, German).
- Dedieu H., Kennedy M. P. & Hasler M. [1993] "Chaos shift keying: modulation and demodulation of a chaotic carrier using self-synchronizing Chua's circuits," *IEEE Trans. Circuits Syst. II* **40**, 634–642.
- Kolumbán G., Vizvari B., Schwarz W. & Abel A. [1996] "Differential chaos shift keying: a robust coding for chaotic communication," *Proc. NDES* **1**, pp.87–92.
- Kolumbán G., Kennedy M. P., Kis G. & Jako Z. [1998] "FM-DCSK: a novel method for chaotic communications," *Proc. IEEE International Symposium on Circuits and Systems* **1**, pp.477–480.
- Kennedy, M. P., Kolumbán, G., Kis, G. & Jako, Z. [2000] "Performance Evaluation of FM-DCSK Modulation in Multipath Environments," *IEEE Trans. Circuits Syst.-I* **47**, 1702–1711.
- Xia, Y., Tse, C. K. & Lau, F.C.M. [2004] "Performance of Differential Chaos-Shift-Keying Digital Communication Systems Over a Multipath Fading Channel With Delay Spread," *IEEE Trans. Circuits Syst.-II* **51**, 680–684.
- Ye, L., Chen, G. & Wang, L. [2005] "Essence and Advantages of FM-DCSK Technique versus Traditional Spreading Spectrum Communication Method," *J. Circuits, Systems Signal Processing* **24**, 657–673.
- Yao, J. & Lawrence, A. J. [2006] "Performance Analysis and Optimization of Multi-User Differential Chaos-Shift Keying Communication Systems," *IEEE Trans. Circ. Syst.-I* **53**, 2075–2091.
- Wang, L., Zhang, C., & Chen, G. [2008] "Performance of an SIMO FM-DCSK Communication System," *IEEE Trans. Circuits Syst.-II* **55**, 457–461.
- Rushforth, C. [1964] "Transmitted-reference techniques for random or unknown channels," *IEEE Trans. Inform. Theory* **10**, 39–42.
- Kolumbán, G. [2000] "Theoretical Noise Performance of Correlator-Based Chaotic Communications Schemes," *IEEE Trans. Circuits Syst.-I* **47**, 1692–1701.
- Kolumbán, G. [2005] "UWB technology: chaotic communications versus noncoherent impulse radio," *Proc. European Conf. Circuit Theory Design* **2**, pp. II 79–82.
- Chong C. C. & Yong S. K. [2008] "UWB Direct Chaotic Communication Technology for Low-Rate WPAN Applications," *IEEE Trans. Vehicular Technology* **57**, 1527–1536.
- Min, X., Xu W. K., Wang, L. & Chen, G. [2010] "Promising Performance of an FM-DCSK UWB System under Indoor Environments," *J. IET Communications* **4**, 125–134.
- Andrews J. G., Ghosh A. & Muhamed R. [2007], "Fundamentals of WiMAX, Understanding Broadband Wireless Networks," Prentice Hall.
- Kaddoum G., Charge P. & Roviras D. [2009] "A generalized methodology for bit-error-rate prediction in correlation-based communication schemes using chaos," *IEEE Communications letters* **13**, 567–569.