

# Designing Delay Lines Based on the SD/DE Algorithm for Transmitted-Reference Ultra-Wideband Systems

Zhexin Xu • Lin Wang • Guanrong Chen<sup>1</sup>

**Abstract** This paper studies the design problem of delay lines in Transmitted-Reference Ultra-Wideband (TR-UWB) systems based on a new evolutionary scheme named Stepping-Dichotomy/Differential-Evolution (SD/DE) algorithm. The group delay response, required for the pass-band as an objective of the delay line, is approximated by using a new nonlinear optimization method under a maximin criterion. The procedure is rather generic, applicable to arbitrary bandwidths, delays and distortions. Moreover, the approach is very convenient for calculating the least-order delay line, therefore especially suitable for real applications.

**Keywords** Delay line • Maximin criterion • Stepping Dichotomy • Differential Evolution algorithm • TR-UWB system

## 1 Introduction

The transmitted reference ultra-wideband (TR-UWB) system was proposed in [14], with a simple receiver structure that captures all energies without requiring channel estimation. Due to the frame structure of the TR-UWB system, the signal received at the receiver end is delayed by the same semi-frame duration for demodulation using a delay line (DL). The group delay characteristic of DL at the receiver end directly influences the performance of the entire system [1, 20, 29]. Therefore, how to find an effectively opera-

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tional method to design an accurate wideband analog DL for demodulation becomes an important technical problem for many related real-world applications.

A perfect delay line should keep a fixed delay time but not change the magnitude or shape of the input signal. The magnitude response of an all-pass filter (APF) is ideally constant for all frequencies and the all-pass group delay function is useful for approximating a specified delay characteristic. Hence, APF is an advisable candidate for the task. Any APF can be decomposed into a cascade of first- and second-order sub-filters. It was noted in [3] that the group delay response of the first-order APF is a decreasing function, so it is only suitable to be used for low-pass DL. On the other hand, the second-order APF has a delay function with the required band-pass characteristic and is regarded as the product of two special first-order APFs. Therefore, the second-order APF is a key component of the delay line structure and hence is the objective of our new design in this paper, which operates within a nanosecond group delay and occupies the bandwidth of the TR-UWB system.

Technically, the design of a required group delay is a typical mathematical approximation problem, for which the maximally flat, least-square and equiripple approximations are well-known methods to apply [3]. In particular, the Bessel-Thomson approximation based on Bessel polynomials [18] is widely used, which however leads to a low-pass delay function. Padé approximation [2, 16, 17], on the other hand, is a common approach using a truncated Taylor expansion of the ideal transfer function around one point. The main drawbacks of these common methods are: (1) they often depend on the designer's intuition or experience, thus do not provide a generic procedure, for which actually a trial-and-error process is unavoidable in most designs; (2) the designer has to reconstruct the whole hardware architecture once there are any changes in design parameters such as the order parameter.

The aforementioned mathematical approximation problems can be cast into optimization problems, which however are often highly nonlinear, non-differentiable, and with multiple minima. Nevertheless, various evolutionary algorithms are powerful techniques for solving these problems, including for instance the Nelder–Mead algorithm (NMA) [6, 8], Particle Swarm Optimizer (PSO) [9, 13] and Genetic Algorithm (GA) [19, 28]. The performances of the above-mentioned methods are generally excellent, but they lack of robustness due to their computational complexity and their common setting with a large number of undetermined parameters. Differential Evolution (DE) algorithm proposed by Storn and Price in [24], on the other hand, is an excellent evolutionary type of algorithm for solving optimization problems involving real-valued functions. DE follows a simple and straightforward logic and has only a few algorithm parameters to tune. Moreover, the high convergence characteristics and robustness of DE makes it a reliable and versatile function optimizer. It recasts the transfer function approximation to a minimization problem of the error function. For these reasons, DE is widely used today [7, 11, 12] with various improved versions developed [3, 4, 22]. Its best version today is a completely parallel search scheme with an unconstrained penalty function, which does not require the functional continuity or any predefined probability density function.

Taking into account the above considerations, this paper develops a novel method, called Stepping-Dichotomy/Differential-Evolution (SD/DE) algorithm, for designing analog DLs for TR-UWB systems. The new scheme is built on the superposition of group delay curves under a maximin criterion, by embedding the standard DE algorithm into the original SD algorithm. The main reasons for using the standard DE algorithm are: (1) it is easy to implement, since it has only three parameters while the other modified versions have more; (2) complex DEs are more suitable for high-dimensional applications [4, 5, 22], but for low-dimensional problems they may lead to slower convergence without much improvement on the optimal solution. Therefore, a relatively simple version, named *DE/best/1/b* in [26], is adopted in this paper.

SD/DE is more effective than DE and is deemed practical for designing DLs because it can find the optimal order in addition to searching for optimal values of the free parameters. In the algorithm, a reasonable all-pass transfer functional format is chosen with some new variables [30]. Then, a proper objective function that reflects the restriction of the DL is designed. To that end, the SD algorithm is used to find the optimal order of the DL while the DE algorithm is applied to minimize the objective function by searching optimal values of the free parameters in the transfer function. The algorithm is rather generic, applicable to arbitrary bandwidths, delays and distortions. The effects of the SD/DE algorithm will be compared to the traditional methods such as Bessel-Thomson approximation and Padé approximation. Simulation results will be given to show that it is convenient for calculating the least-order DL. The DL model proposed here can be regarded as the cascade of several second-order sub-filters, which can be designed separately. Therefore, designing DL in the new way is no more complex than that of a second-order one, and this clearly shows its distinct superiority suitable for many real-life applications. Moreover, NMA and PSO will be embedded respectively into SD to form SD/NMA and SD/PSO in order to be comparable with SD/DE. The result shows that the system order, time consumption in program-running and the iteration number of SD/DE are less than the others. It proves the effectiveness of the DE in solving this kind of problems.

The rest of the paper is organized as follows. The new SD/DE algorithm is described in Section II, with simulation results presented in Section III. Conclusions are drawn in Section IV.

## **2 Transfer Function of the Delay Line**

As mentioned, the all-pass transfer function is an appropriate choice. Design of DL will become simpler if some parameters are introduced to adjust for a specific group delay response. Besides, first-order and second-order all-pass functions are important, for any all-pass characteristic can be realized as a product of these two simple forms. Moreover, because the delay function of the second-order all-pass filter rather than the first-order one has a band-pass shape, it is useful for band-pass systems such as the

TR-UWB system in interest.

From the above perspective, consider the  $2n^{\text{th}}$ -order transfer function denoted by cascading second-order APFs:

$$H_{total}(s) = \prod_{i=1}^n H_i(s) = \prod_{i=1}^n \frac{s^2 - \frac{wn_i}{Q_i} s + wn_i^2}{s^2 + \frac{wn_i}{Q_i} s + wn_i^2} \quad (1)$$

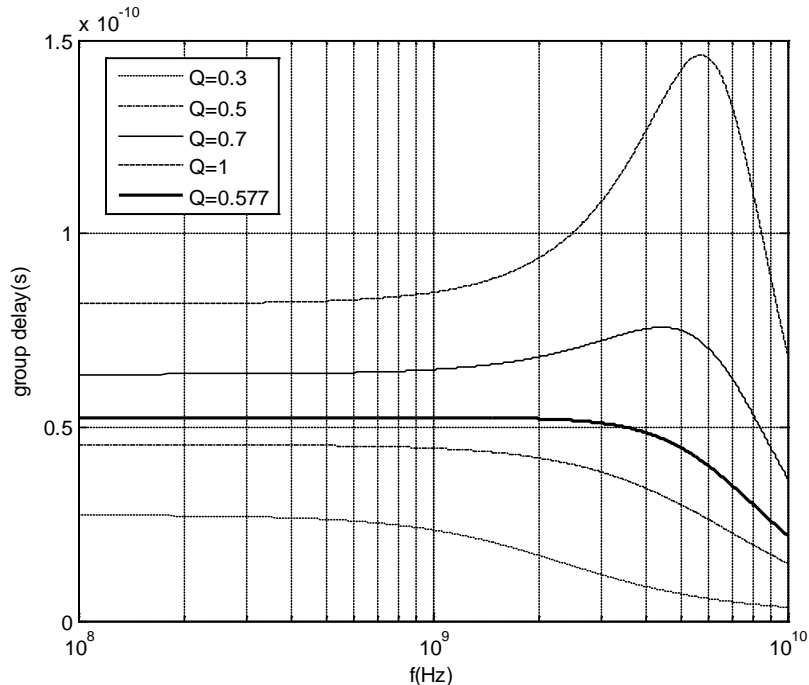
In the sub-filter transfer function  $H_i(s)$ ,  $wn_i$  is the natural frequency and  $Q_i$  is a quality factor; they together determine the position of poles of the transfer function in the complex plane. The group delay of this system is given by (2) [30]

$$D_{total}(w) = \sum_{i=1}^n D_i(w) = \sum_{i=1}^n \frac{2}{Q_i \cdot wn_i} \cdot \frac{1 + \left(\frac{w}{wn_i}\right)^2}{\left(1 - \left(\frac{w}{wn_i}\right)^2\right)^2 + \left(\frac{w}{Q_i \cdot wn_i}\right)^2} \quad (2)$$

This function is composed of  $D_i(w)$ , which denotes the group delay of  $H_i(s)$ . Without loss of generality,  $D(w)$  is defined by

$$D(w) = \frac{2}{Q \cdot wn} \cdot \frac{1 + \left(\frac{w}{wn}\right)^2}{\left(1 - \left(\frac{w}{wn}\right)^2\right)^2 + \left(\frac{w}{Q \cdot wn}\right)^2} \quad (3)$$

The variables  $wn$  and  $Q$  control the shape of  $D(w)$ , as shown in Fig. 1, which are obtained from (3) by setting  $wn = 7e9 * 2\pi \text{ rad} / s$  and  $Q = 0.3, 0.5, 0.577, 0.7, 1$ , respectively. One can find a similar conclusion in [30].



**Fig. 1** Group delay of second-order all-pass function with different  $Q$  values, where  $wn = 7e9 * 2\pi \text{ rad} / s$

As is shown in Fig.1, for  $Q > 1/\sqrt{3}$ , e.g.,  $Q = 0.7$  and  $Q = 1$ , the delay functions have a narrow band-pass shape with a peak value  $D_{max}$  at  $wn_{peak}$ , yielding

$$wn_{peak} = wn\sqrt{-1 + \sqrt{4 - Q^{-2}}} \quad (4)$$

and  $D_{max}$  is calculated according to

$$D_{max} = \frac{2Q}{wn_{peak}} \cdot \frac{A^{1/2}}{A^{3/2} - 2QA} \quad (5)$$

where

$$A = 4Q^2 - 1 \quad (6)$$

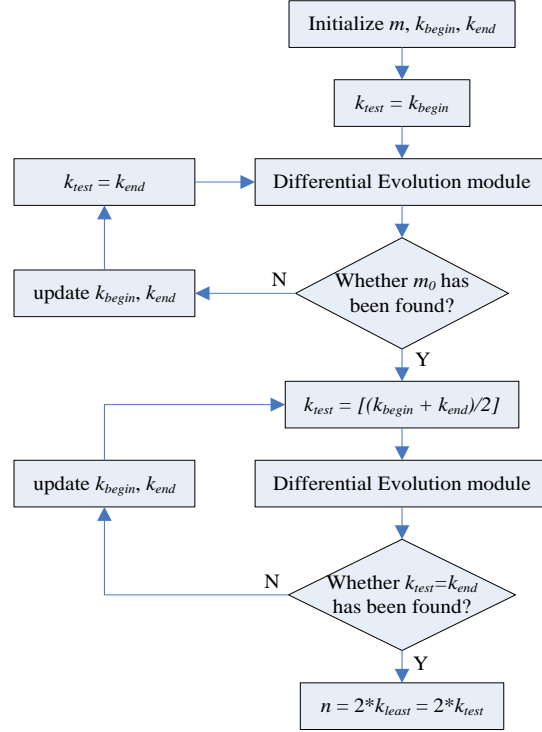
As  $Q$  decreases,  $D_{max}$  drops and  $wn_{peak}$  moves towards 0 while the delay functions are broaden. Finally, at  $Q = 1/\sqrt{3} \approx 0.577$ ,  $D_{max}$  takes places at  $wn_{peak} = 0$  and the maximal flat delay is achieved. For  $Q < 1/\sqrt{3}$ , e.g.,  $Q = 0.3$  and  $Q = 0.5$ , the maximum delay decreases with a smaller  $Q$ . The delay-bandwidth-product decreases as well. Especially, the delay for  $Q = 0.5$  is twice the delay of the corresponding first-order function, so it can be treated as a product of two first-order APFs. Considering the pass band of the TR-UWB system, the desired correction is a delay function with  $Q > 1/\sqrt{3}$ .

A reasonable superposition of these curves will achieve the required group delay characteristic through controlling the values of  $Q_i$  and  $wn_i$ , because each  $D_i(w)$  corresponds to a group delay curve.

### 3 Stepping-Dichotomy/ Differential-Evolution Algorithm

In general, the performance of the receiver is acceptable if the group delay characteristic of the DL has only a small distortion, such as  $T_d \pm \delta$ , where the variable ripple  $\delta$  denotes the allowable maximal deviation from the semi-frame duration  $T_d$ . So, the group delay of DL should be designed to meet the requirement on  $T_d$  and  $\delta$ . First, a reasonable transfer function  $H(s)$  of the DL is chosen, as mentioned above. Then, a proper objective function that reflects the restriction of the DL is designed. To that end, the SD algorithm is used to find the optimal order of the DL, after computing a lower bound of the number of the group delay curves. Meanwhile, the DE algorithm is applied to minimize the objective function by searching for optimal values of the free parameters in  $H(s)$ .

A flowchart of this new algorithm is presented in Fig. 2, with details to be described below, using  $n = 2k$  for convenience.



**Fig. 2** Flowchart of the Stepping-Dichotomy / Differential-Evolution Algorithm

The SD is used to find the least order of the DL. Here, the order  $n$  is indirectly reflected by the parameter  $k$ , which denotes the number of group delay curves for superposition.

*Step 0:* The initial  $k^{(0)}$  value is set to be 1. (It means that at least one curve is needed.)

*Step 1:* Choose  $k^{(0)}$  as the initial curve number. It can be judged whether the superposition of  $k_{test} = 2^{m-1}k^{(0)}$  curves fulfill the requirement by cumulating up to an integer  $m$  ( $m=1,2,\dots$ ). This process is accomplished when we find  $m_0$  which results in  $2^{m_0}k^{(0)}$  curves become feasible but  $2^{m_0-1}k^{(0)}$  curves do not. Thus, the target interval, in which the least curve number  $k_{least}$  lies, can be determined from  $(2^{m_0-1}k^{(0)}, 2^{m_0}k^{(0)})$ . Moreover,  $k_{begin}$  and  $k_{end}$  can be defined as the initial point and ending point of the interval, respectively, where

$$\begin{aligned} k_{begin} &= 2^{m_0-1}k^{(0)} \\ k_{end} &= 2^{m_0}k^{(0)} \end{aligned} \quad (7)$$

*Step 2:* Test the point  $k_{test}$  of the region  $(2^{m_0-1}k^{(0)}, 2^{m_0}k^{(0)})$ . Here,  $k_{test}$  is computed according to

$$k_{test} = \left\lceil \frac{k_{begin} + k_{end}}{2} \right\rceil \quad (8)$$

where  $[a]$  is the nearest integer greater than or equal to the real number  $a$ . If  $k_{test}$  is not feasible, then  $k_{least} \in (k_{test}, k_{end}]$ , so update  $k_{begin}$  by  $k_{test}$ ; otherwise,  $k_{least} \in (k_{begin}, k_{test}]$ , so update  $k_{end}$  by  $k_{test}$ .

*Step 3:* Repeat *Step 2* until  $k_{test}$  is equal to  $k_{end}$ . Now, one obtains the least curve number  $k_{least}$  to superpose the required group delay:

$$k_{least} = k_{test} = k_{end} \quad (9)$$

Thus, the least order of DL is  $n = 2 k_{least}$ .

The above SD algorithm is designed based on the following reasons. If an objective curve can be superposed by  $k$  curves, it can be superposed by more than  $k$  curves; whereas it cannot be superposed by less than  $k$  curves if an objective curve cannot be superposed by  $k$  curves.

There is a crucial issue that should be addressed; that is, how can one judge whether the superposition of  $k_{least}$  curves fulfill the requirement? This problem is solved by modifying the DE algorithm, as further discussed below.

As the least curve number  $k_{least}$  is obtained, (2) should be updated as

$$D(w, \{wn_i\}_1^{k_{least}}, \{Q_i\}_1^{k_{least}}) = \sum_{i=1}^{k_{least}} D_i(w, \{wn_i\}_1^{k_{least}}, \{Q_i\}_1^{k_{least}}) = \sum_{i=1}^{k_{least}} \frac{2}{Q_i \cdot wn_i} \cdot \frac{1 + \left(\frac{w}{wn_i}\right)^2}{\left(1 - \left(\frac{w}{wn_i}\right)^2\right)^2 + \left(\frac{w}{Q_i \cdot wn_i}\right)^2} \quad (10)$$

The free parameters that should be optimized in this equation are  $wn_i$  and  $Q_i$ ,  $i = 1, 2, \dots, k_{least}$ . The DL shall be designed by using a maximin criterion via minimizing an objective function, defined by

$$F(\vec{x}) = \max(\text{dev}(D(w))) + P \quad (11)$$

where

$$\text{dev}(D(w)) = |D(w) - T_d| \quad (12)$$

and

$$\vec{x} = (x_1, x_2, \dots, x_{2k_{least}}) \quad (13)$$

with mapping

$$x_i = wn_i, \quad i = 1, 2, \dots, k_{least} \quad (14)$$

$$x_i = Q_{i-k_{least}}, \quad i = k_{least} + 1, k_{least} + 2, \dots, 2k_{least} \quad (15)$$

The major part of the objective function,  $\max(\text{dev}(D(w)))$ , denotes the maximum absolute deviation of the group delay from  $T_d$ , which denotes the semi-frame duration. Otherwise, the objective function should contain some penalties that reflect the constraints

of DL. Here, DL must be steady and realizable, so that the poles of the transfer function lie in the left-hand plane while the zeros in the right-hand plane. Therefore, the constraints are

$$\begin{cases} wn_i > 0 \\ Q_i > 0, \end{cases} \quad i = 1, 2, \dots, k_{least} \quad (16)$$

The penalty term  $P$  in (11) is used to define the domain of independent variables,  $wn_i$  and  $Q_i$ , which can be calculated via

$$P = \sum_{i=1}^{k_{least}} \begin{cases} 20000 - 100 \cdot x_i, & x_i < 0 \\ 0, & otherwise \end{cases} \quad (17)$$

If  $x_i$  is out of the domain,  $P$  should be larger obviously than the value when  $x_i$  is reasonable. So, in (17), set  $P = \sum_{i=1}^{k_{least}} (20000 - 100 \cdot x_i)$  when  $x_i < 0$ .

Practically, the objective function is only applied at a finite number of points in the frequency domain, e.g.,  $N$  equidistant points in the pass-band.

The vector  $\vec{x}$  that minimizes (11) is a solution of the DL design problem. A nonlinear optimization algorithm is required to work out the global minimization of (11), which is a highly nonlinear function with multiple modals. The Differential-Evolution (DE) scheme has proven a promising candidate as it is simple to understand and to implement. Some control variables exist in DE to determine the convergence properties.

By nature, DE is a parallel direct search strategy for minimization with iterations, which is described as in [24]. It starts by initializing the population, which refers to a group of  $NP$   $D$ -dimensional parameter vectors, distributed randomly under a uniform probability distribution within the domain of these vectors. Here, the dimension  $D$  is equal to the number of free parameters. Then, the perturbed parameter vectors are determined under according to the weight  $F$ , randomly chosen from the initial population. Further, trial vectors are generated during the crossover operation with crossover rate  $CR$ . Finally, trial vectors are compared with parameter vectors. If the fitness value of a trial vector is lower than that of the parameter vectors, then the trial vector will become a member of the next generation; otherwise, the parameter vector is chosen. The optimization process is stopped when a budget condition is reached

In the DL design, it is necessary to modify the basic DE scheme.

Firstly, because of the effects of  $wn$  and  $Q$  on controlling the shape of the group delay curve, initial population is generated via

$$P_0 = \{x_{1,0}, x_{2,0}, \dots, x_{k_{least},0}, x_{k_{least}+1,0}, x_{k_{least}+2,0}, \dots, x_{2k_{least},0}\} \quad (18)$$

where

$$x_{j,i,0} = wn_{j,0} = w_{lo} + rand_j[0,1] \cdot (w_{up} - w_{lo}), \quad j = 1, 2, \dots, k_{least} \quad (19)$$



and

$$x_{j,i,0} = Q_{j-k_{least},0} = 1/\sqrt{3} + rand_j[0,1] \cdot (Q_{\max,i} - 1/\sqrt{3}), \quad j = k_{least} + 1, k_{least} + 2, \dots, 2k_{least} \quad (20)$$

Here,  $w_{lo}$  and  $w_{up}$  denote the lower and upper cutoff frequencies, respectively. This is the reason for choosing (1) as the transfer function of DL.

Secondly, DE/*best/1/bin*, a variant of DE [26], is chosen for it has been proved more effective than the basic DE scheme.

Finally, the termination condition for iteration is set, which should be done specifically. In the basic DE, iteration is ended when the objective function achieves the requirement or the maximal number of iterations is reached. However, the curve number provided by SD may not be enough to achieve the requirement in case the lower bound  $k^{(0)}$  is not tight. So, one should find the global minimum, but not the maximal iteration number, and use it to check whether the curve number is appropriate.

To summarize, the termination condition in the new design is when the absolute deviation from  $T_d$  is less than  $\delta$  with the optimal result or the value of the objective function does not change for several generations, which is regarded as achieving the global minimum.

### 3 Simulations and Analysis

To illustrate the performance of the proposed approach, let us design a DL in the TR-UWB system with band-pass spanning 3 to 4GHz and frame duration of 4ns, thus  $T_d$  should be 2ns. Variable ripple  $\delta$  is assumed to be 0.02ns.

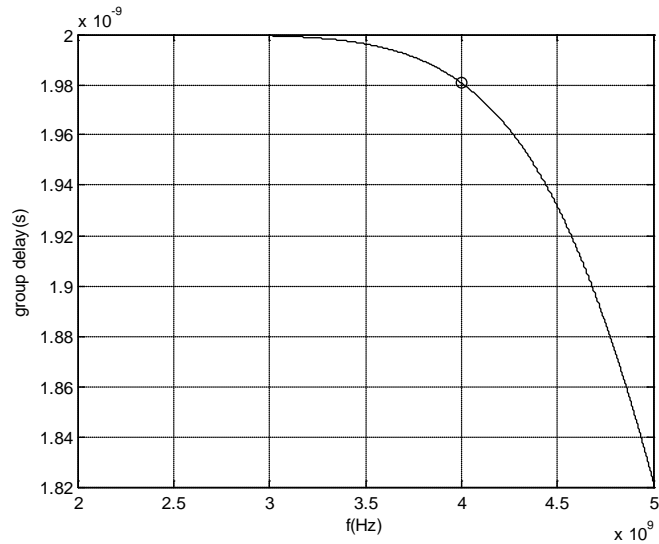
#### 3.1 Compare with traditional methods

First, it is to compare the effect of the SD/DE algorithm with the traditional mathematical approximations. Here, Bessel-Thomson approximation and Padé approximation are taken as examples. Bessel-Thomson approximation is the most widely used one in designing delay filters, which gives an approximately constant time delay over a large frequency range [27, 18]. The transformation function has the following form:

$$H(s) = \frac{\theta_n(0)/T_d}{\theta_n(s/w_0)} \quad (21)$$

where  $\theta_n(s)$  is a reverse Bessel polynomial, from which the filter gets its name, and  $w_0$  is a frequency chosen to give the desired cut-off frequency. The filter has a low-frequency group delay,  $T_d$ . To achieve the same goal of design, the transformation function is set with  $n = 8$  in order to perform a fair comparison with SD/DE. So, the only parameter needed to be determined is  $w_0$ . The cut-off

frequency is obtained as  $\omega_0 = 2\pi f_0 = 4.8356e9 \text{ rad/s}$ . The corresponding group delay response is illustrated by Fig. 3, which shows that the maximal distortion of group delay in the pass-band is 0.01910119ns.



**Fig. 3** Group delay response of the Bessel-Thomson filter with  $n = 8$  and  $\omega_0 = 2\pi f_0 = 4.8356e9 \text{ rad/s}$

An advantage of the Bessel-Thomson approximation is that there are fewer parameters than SD/DE. Besides, it is easy to meet the requirement of group delay response in the pass-band, but the stop-band performance is difficult to control. However, the optimal values of parameters need be chosen based on the designer's intuition or experience. So, the main problem in real applications is that the design is a trial-and-error process.

Padé approximation, on the other hand, is a method of approximating the objective function by concentrating around one point in the form of a ratio of two power series [2]. The coefficients of the approximating rational expression are computed from the Taylor coefficients of the original function. If the numerator has order  $m$  and the denominator has order  $n$  in the approximation rational function, then one can approximate the original function up to order  $m+n$ . The Padé approximant often gives a better approximation of the function than its simple truncated Taylor series approximation, and it may still work even if the Taylor series method does not converge. The reason to apply Padé approximation is that it immediately yields a rational expression, which is convenient for implementation [15, 16]. Here, the method is applied in the Laplace transformed domain and the objective function is  $H(s) = e^{-T_d s}$ .

The rational function of Padé approximation at  $s = s_0$  is expressed as follows:

$$\hat{H}_{m,n}(s) = \frac{P_m(s-s_0)}{Q_n(s-s_0)} = \frac{\sum_{i=0}^m p_i \cdot (s-s_0)^i}{\sum_{j=0}^n q_j \cdot (s-s_0)^j} \quad (22)$$

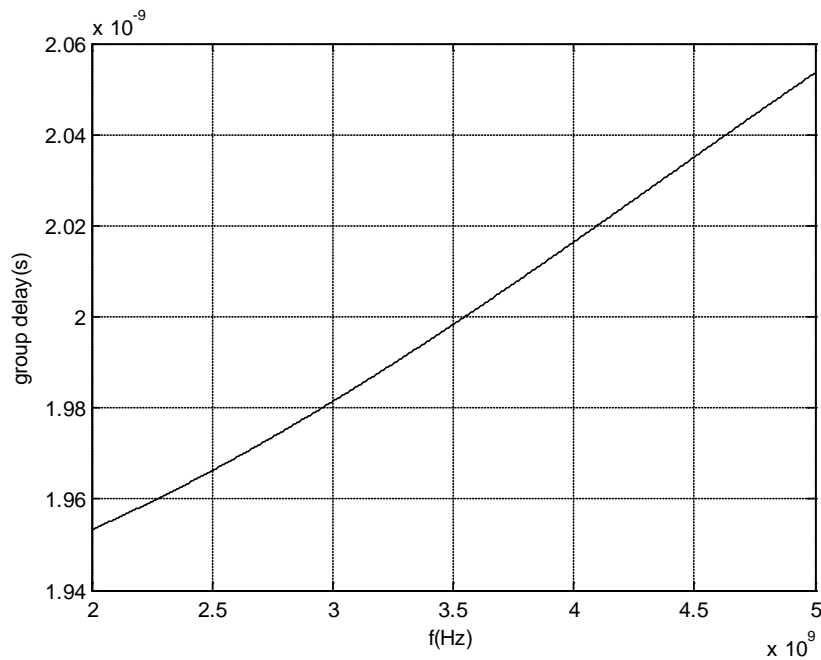
The coefficients of  $P_m(s)$  and  $Q_n(s)$ , which are determined by the first  $(m+n+1)$  coefficients of the Taylor series, can be calculated following [16]. And, then, the result is transformed into the Fourier domain, as

$$\hat{H}_{m,n}(w) = \frac{P_m(w/w_e - s_0)}{Q_n(w/w_e - s_0)} = \frac{\sum_{i=0}^m p_i \cdot (w/w_e - s_0)^i}{\sum_{j=0}^n q_j \cdot (w/w_e - s_0)^j} \quad (23)$$

where  $w_e = 54$  is a expansion factor,  $s_0 = 2.7e9$ , and

$p_0 = 0.004516580942613$	$q_0 = 1$
$p_1 = -1.224275206492067e-11$	$q_1 = -7.106238591704979e-10$
$p_2 = 1.671707325177463e-20$	$q_2 = 2.800195597573023e-19$
$p_3 = -1.532731972612903e-29$	$q_3 = -7.894675938903919e-29$
$p_4 = 8.805863541963319e-39$	$q_4 = -3.824230954441388e-37$
$p_5 = -3.225103069643869e-48$	$q_5 = -2.072358655257696e-46$
$p_6 = 7.635204476528408e-58$	$q_6 = -5.090785117443841e-56$
$p_7 = -1.115348810779172e-67$	$q_7 = -6.069524006345791e-66$
$p_8 = 8.053960800980580e-78$	$q_8 = -2.457972659915661e-76$

The corresponding group delay response is illustrated in Fig.4, which shows that the maximal distortion of group delay in pass-band is  $1.868125e-011$ ns.



**Fig. 4** Group delay response of Padé approximation with  $m=n=8$ ,  $w_e = 54$ ,  $s_0 = 2.7e9$

The limitation of this method includes (see [17]): (1)  $s_0$  should be selected properly to trade off the stability and the good fit around this point in the time domain; (2) an unlucky choice of the numerator order  $m$  and/or the denominator  $n$  may lead to an

inconsistent system of equations or an unstable approximation. Changing  $m$  or  $n$  also depends on the designer's experience just as choosing the order in the Bessel-Thomson approximation.

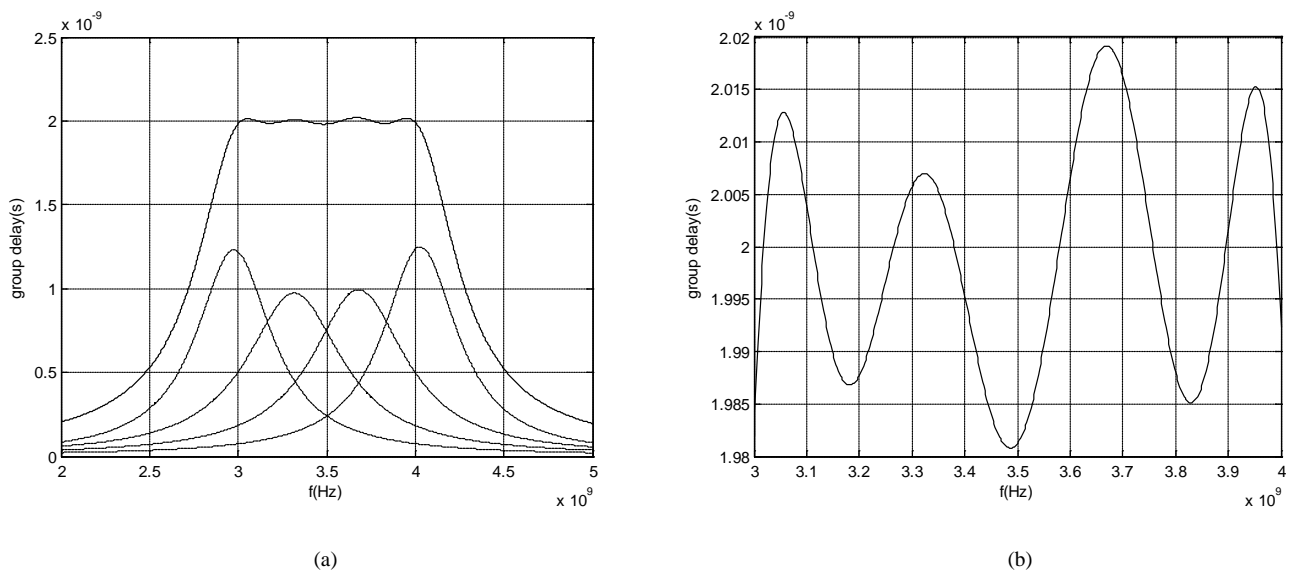
A DL can be designed using the SD/DE algorithm as long as  $T_d$ ,  $\delta$  and the pass-band of the TR-UWB system are enacted. Control variables of DE are set as  $NP=30$ ,  $CR=0.8$ ,  $F=0.8$ . The objective function will converge quickly only when the control variables in DE such as  $NP$ ,  $F$  and  $CR$ , are all correctly set. The issue as how to choose them was already raised in [26]. Totally 100 equidistant points are sampled from inside the pass band.

Simulation results using MATLAB are shown in Fig. 5(a). The detailed distortion in pass-band 3 to 4GHz is shown in Fig. 5(b). The results show that the maximal distortion is 0.019327ns from 2ns in the required pass-band, which means that the superposition of curves achieves the expected agreement. Using the SD/DE algorithm with the aforementioned parameters setting, DL is designed after only 1634 generations. 4 curves is found to be the optimal number eventually. The corresponding optimal parameter values are given by  $wn_i$  and  $Q_i$  ( $i=1,2,3,4$ ). The transfer function is obtained according to

$$H(s) = \prod_{i=1}^4 \frac{s^2 - \frac{wn_i}{Q_i} s + wn_i^2}{s^2 + \frac{wn_i}{Q_i} s + wn_i^2} \quad (24)$$

where

$$\begin{aligned} wn_1 &= 2.3208e10, & Q_1 &= 5.7554 \\ wn_2 &= 2.0944e10, & Q_2 &= 5.0838 \\ wn_3 &= 2.5358e10, & Q_3 &= 7.9092 \\ wn_4 &= 1.8770e10, & Q_4 &= 5.7652 \end{aligned}$$



**Fig. 5** Group delay frequency responses of DL. (a) the response in 2GHz-5GHz. Dashed line shows the group delay behavior of sub-DLs with  $k_{leas}=4$  and real line is the one of total DL. (b) the response in 3GHz-4GHz

Comparing to the aforementioned traditional methods, the main advantage of the SD/DE algorithm is that it can select the optimal order of the system automatically. In addition, the requirements of group delay response in both pass-band and stop-band can be easily achieved by using the objective function, which contains some penalties that reflect the constraints in the SD/DE algorithm. Even more importantly, the hardware architecture of the results from SD/DE has better expansibility than the aforementioned traditional methods, because it can be regarded as a cascade of several second-order sub-filters, which can be designed separately. Therefore, the design of DL is not more complex than that of a second-order one, showing its distinct superiority for many real applications.

### 3.2 Comparison of the effects of various methods embedding other evolutionary algorithms into SD

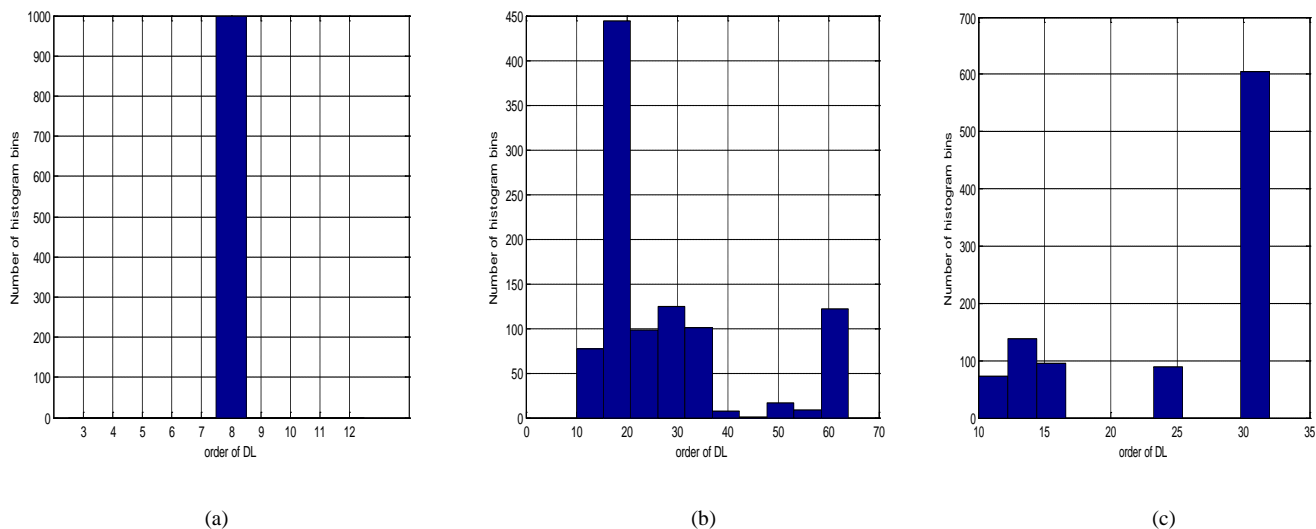
To further show the effectiveness of the proposed SD/DE algorithm, other evolutionary algorithms such as NMA and PSO are embedded into SD for comparison. The performances of SD/DE, SD/NMA and SD/PSO are compared for the design of the above-described DL. For each algorithm, 1000 independent runs were performed on a PC with Intel Core(TM)2 Duo E4500 at 2.20GHz with 2 GB RAM running Windows 7, to characterize their computational complexity and effectiveness.

Control variables of DE are set as  $NP = 30$ ,  $CR = 0.8$ ,  $F = 0.8$ . A total of 650 generations were used in testing the  $k_{test}$  curves, so this number is large enough to find the optimal results. NMA is used in the process as described in [21]. A sufficient maximum number of iterations is  $2e4$  for each  $k_{test}$ . PSO is adopted following the instructions given in [10]. The control parameters of PSO are set as follows: the number of particles is 30, the number of maximum iterations is 1000 (since 650 is not large enough to find the optimal solution here). Moreover, just like DE, the initial population is generated randomly.

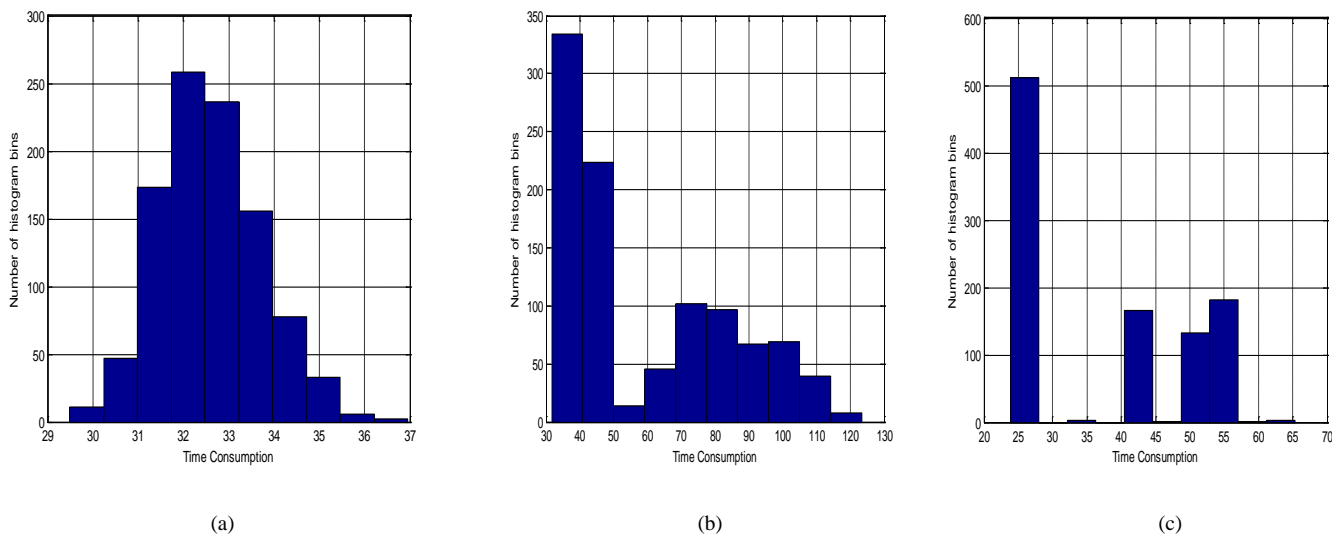
Fig. 6 shows the histograms of the optimal order of the delay line, calculated by SD/DE, SD/NMA and SD/PSO, respectively, all over 1000 runs. It can be observed that the optimal order found by SD/DE is 8, which is less than that found by the other methods. It means that the DL designed using SD/DE has lower complexity. In addition, in Fig. 6, which shows the distribution of the optimal order calculated by the three methods, it can be seen that the order of SD/DE is at the constant concentration of 8, while the others are distributed over 10 to 60. So, DE outperforms the others on finding the global minimum of the fitness function with multiple minima, and it has better robustness as well. Clearly, even when  $k_{test}$  is sufficiently large, NMA and PSO often converge to local minima, which do not meet the requirement in general, so it is incorrect to assume they need larger values of  $k_{test}$ .

The above reasoning can also be used to explain the phenomena reflected by Fig. 7, which shows the histograms of time consumptions, and by Fig. 8, which shows the histograms of the iteration number. The poorer judgments of NMA and PSO result in larger  $k_{test}$  which then leads to more time consumption and a larger number of iterations. The mean time consumption of SD/DE is

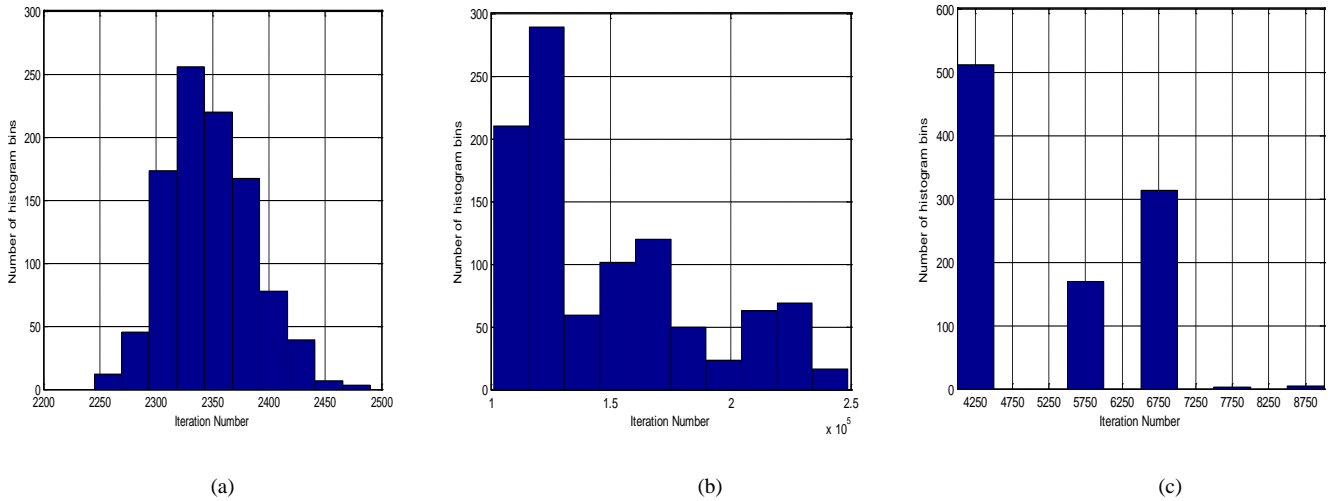
32.5995s, less than that of SD/NMA which is 60.825s and that of SD/PSO which is 38.908s. Moreover, the mean iteration number of SD/DE is 2384.2, also less than that of SD/NMA which is 148850 and that of SD/PSO which is 5423.75. So, all these together have proved that SD/DE is faster and more efficient than the others in designing a delay line.



**Fig. 6** The histograms of optimal order calculated by SD/DE, SD/NMA, SD/PSO, respectively, all for 1000 times. (a) SD/DE (b) SD/NMA (c) SD/PSO



**Fig. 7** The histograms of time consumptions employed SD/DE, SD/NMA and SD/PSO, respectively, all for 1000 times. (a) SD/DE (b) SD/NMA (c) SD/PSO



**Fig. 8** The histograms of iteration numbers employed SD/DE, SD/NMA and SD/PSO, respectively, all for 1000 times. (a) SD/DE (b) SD/NMA (c) SD/PSO

Now, consider the effects of other parameters in this design. First, it is obvious that the larger the  $T_d$  or the wider the pass-band, the more curves are needed to fill up a much larger area. Also, a smaller  $\delta$  requires more curves, since the curves should be arranged closer to each other while meeting the requirement of  $T_d$ . Moreover, the objective function is only approximated at the sampled points. Hence, enough number of points is needed to ensure the curve superposition achieve the requirement in the whole pass-band. Nevertheless, this does not mean the more sample points the better approximation effect, because extra points are useless for improving the distortion but only increasing the amount of calculation. Generally, 100 to 200 points per GHz should be enough. Finally, in this design, the sampled points span equidistantly but they are more practical in un-equidistant mode; namely, the sampled points should be dense in the frequency range with higher power, but sparse otherwise, so that the total point number may be significantly reduced. It is remarked that the solution is not unique since DE depends on the stochastic kernel. But, all the solutions are effective.

#### 4 Conclusions

The design problem of an analog delay line for a TR-UWB system has been investigated through a novel procedure different from conventional mathematical approximations. An effective design methodology, called Stepping-Dichotomy/Differential-Evolution (SD/DE) algorithm, has been proposed, which is a nonlinear optimization scheme by nature. The required group delay response is approximated under a maximin criterion in the pass-band of the TR-UWB system. This algorithm is much more effective and indeed more practical compared to the other conventional ones on optimal order, time consumptions and iteration numbers. The procedure is applicable to arbitrary bandwidths, delays and distortions. More importantly, it is convenient to calculate the least order of DL,

which is the distinct superiority over the others; therefore the new design is suitable for implementing TR-UWB systems in real applications.

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