Designing Delay Lines Based on GDRR for TR-UWB Systems

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Abstract: Designing a non-ideal delay line (DL) with phase distortion in a transmitted-reference ultra-wideband system (TR-UWB) with an autocorrelation receiver (AcR) is a great technical challenge. Differing from the currently empirical design method of DL, in this paper a semi-analytic approach is proposed through Gaussian approximation of the expression for conditional bit error rate (BER), based on investigation on the degradation of average BER caused by a group delay ripple range (GDRR) over independent Nakagami-*m* fading channels. This GDRR-based design method can directly evaluate its effects on the system performance and determine the acceptable phase distortion level to trade off the BER performance and system complexity.

Keywords: transmitted-reference ultra-wideband system, delay line, group delay ripple range, fading channel

1 Introduction

Delay line (DL) is a vital module in transmitted-reference ultra-wideband (TR-UWB) systems with autocorrelation receiver (AcR) because received signals should be delayed by a semi-frame duration when it is used for demodulation. Much effort devoted to analyzing the bit error rate (BER) performance of such systems considered ideal delay lines [1]-[9]. However, there always is some phase distortion resulting in BER degradation in a real DL. So, when designing DL, one needs a good design method to limit the extant of phase distortion, which can be properly characterized by a group delay ripple (GDR) or a group delay ripple range (GDRR). Some investigations [14], [17] on related issues had been carried out and reported, yet no practical design method was suggested in the literature. In this endeavor, simulating the impact of GDR on a system is a relatively simple and direct approach [14]-[16]. But the shortcoming of this method is that the simulation model is formulated only for some very special cases. In other word, one has to reconstruct the simulation model once there are some changes in preconditions. For differential phase-shift keying (DPSK) systems, pulse distortion has been calculated analytically [17], where an analytic expression is derived and it showed how a pulse is affected by the ripple periods and magnitudes, and how the mean signal frequency is located with respect to the ripple peaks. For TR-UWB systems, however, there was no report about theoretical analysis on effects of GDR and GDRR to the best of our knowledge.

Motivated by the above considerations, in this paper we present a theoretical analysis of effects of GDR and GDRR on TR-UWB systems with AcR, which leads to the proposal of a new design method based on Gaussian approximation (GA). We have strong reasons to use GA [1], [2], [4], [5], [9]. Today, GA is still a common and effective method for analyzing the performances of TR-UWB systems [10], [13], although there are other good methods such as the sampling expansion proposed by Quek *et al.* [5], [6] and the Karhunen-Loève (KL) expansion proposed by Niranjayan *et al.* [7], [8]. Since the sampling expansion approach is proved invalid when DL is non-ideal [8] and, comparing to the KL expansion approach, GA is much simpler with repeatedly-verified results fairly close to the real simulations. GA is believed the best choice for this paper.

Our contributions in this paper may be summarized as follows. First, given a group delay ripple, we developed a modified conditional BER. Second, we evaluated the BER performance in a multipath channel by averaging derived expressions. Moreover, we can easily assess the impact of GDR and trade off performance and complexity when designing DL, so that the BER degradation level corresponding to GDRR can be determined by this design method.

The rest of the paper is organized as follows. Section II presents the system model and derives BER expressions based on GA. In order to provide numerical validation of the proposed expressions, Section III presents related simulation results, where independent Nakagami-*m* fading channels will be used. Finally, Section IV concludes the investigation.

2 System Model

Each frame of a TR-UWB symbol contains two pulses. The first pulse is used as a template, whereas the second serves as a data signal which is delayed with respect to the first one by a semi-frame duration T_d. Ideally, the received signal will be delayed by T_d for demodulation [2]. Autocorrelation receiver (AcR) is typically used to exploit the diversity inherent in a multipath channel.

The *i-th* symbol transmitted with N_p pulses can be decomposed into a reference signal block plus a data modulated signal block, as follows:

$$
\begin{aligned}
s_i(t) = \sum_{j=0}^{N_p/2-1} & \sqrt{E_p}\, p\left(t - jT_f - c_jT_c\right) \\
& + b_i\sqrt{E_p}\, p\left(t - jT_f - c_jT_c - T_d\right)
\end{aligned} \tag{1}
$$

where $p(t)$ is the normalized signal pulse with duration T_p, T_f denotes the frame duration, T_c is an additional Time-Hopping (TH) delay, T_d is the delay between the reference pulse and the corresponding data pulse, $E_p = \int_0^{T_p} p^2(t)\,dt$ is the energy of transmitted pulses, c_j is the pseudorandom integer sequence between 0 and N_h, with N_h being the maximum allowable integer shift, and b_j is the *j-th* transmitted data symbol. Take $b_i = 1$ for example. TR signaling is shown in Fig. 1, where solid lines and dashed lines indicate reference pluses and data pluses, respectively.

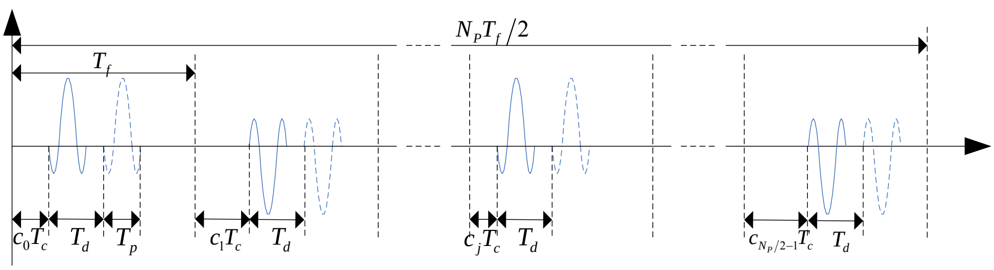

Figure 1 Schematic diagram of TR signaling $s_i(t)$, where $b_i = 1$

The received signal can be expressed as

$$
r_i(t) = \int_{-\infty}^{+\infty} h(\tau)\, s_i(t-\tau)\, d\tau + n(t) \tag{2}
$$

where $h(t)=\sum_{l=1}^{L}\alpha_l\delta(t-\tau_l)$ is the channel impulse response where L is the number of multipath components, α_l is the fading coefficient of the *l-th* path with $E\left[\sum_{l=1}^{L}\alpha_l^2\right]=1$, and τ_l denotes the delay of the *l-th* path.

Let $n(t)$ denote a zero-mean white Gaussian noise with two-sided power spectral density $N_0/2$. The tap delays τ_l are chosen such that $|\tau_l-\tau_j|\geq T_p, \forall l\neq j$, where $\tau_l=\tau_1+(l-1)T_p$, so that the multipath channel can be considered to be resolvable. Furthermore, let the time duration of a received UWB pulse be defined as $T_g=T_m+T_p$, where T_m is the maximum excess delay spread, and let α_l be statistically independent random variables. We preclude intersymbol interference (ISI) and intrasymbol interference by choosing $T_d\geq T_g$ and the bit duration $T_b=2T_f$ to be larger than T_d+T_m.

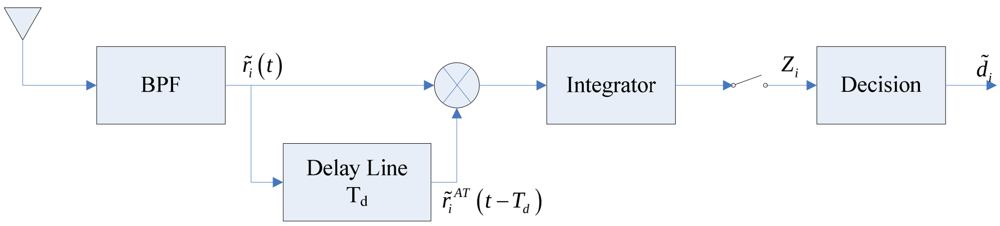

Figure 2 Block diagram of AcR in a TR-UWB system

In an autocorrelation receiver, as shown in Fig.2, the received signal first passes an ideal bandpass filter (BPF) with bandwidth W and center frequency f_c, where W is chosen to be sufficiently wide ($W\gg 1/T_g$), so that when signal passes the ideal BPF, its spectrum will not be distorted. Consequently, the intersymbol and intrasymbol interference caused by BPF are negligible. The *i-th* received symbol after filtering can be written as

$$\tilde{r}_i(t) = \sum_{j=0}^{N_p/2-1} \sum_{l=1}^{L_{CAP}} [\sqrt{E_p}\alpha_l p(t - jT_f - c_j T_c) + b_i \sqrt{E_p}\alpha_l p(t - jT_f - c_j T_c - T_d)] + \tilde{n}(t) \tag{3}$$

where L_{CAP} denotes the number of captured multipath components and $\tilde{n}(t)$ denotes a zero-mean Gaussian random process. The autocorrelation function of $\tilde{n}(t)$ is given by

$$R_{\tilde{n}}(\tau) = WN_0 Sa(W\tau)\cos(2\pi f_c \tau) \tag{4}$$

with $Sa(x) = \sin(\pi x)/\pi x$. Expression (3) can be simplified by applying the relation $g(t) = \sqrt{E_p}\alpha_l p(t) * h_{BPF}(t)$, as follows:

$$\tilde{r}_i(t) = \sum_{j=0}^{N_p/2-1} \sum_{l=1}^{L_{CAP}} [g(t - jT_f - c_j T_c) + b_i g(t - jT_f - c_j T_c - T_d)] + \tilde{n}(t) \tag{5}$$

where $h_{BPF}(t)$ denotes the BPF impulse response. The filtered received-signal passes through a delay line by T_d, so the received data-modulated waveform becomes correlated with the reference one over the integration time T_I ($T_p \le T_I \le T_g$) of the correlator. The amount of multipath captured L_{CAP} is determined by T_I.

2.1 Effect of GDR Function

Ideally, the group delay characteristic of DL is constant, so that all frequency components have equal delay times. Namely, there is no distortion in the range of selected frequencies. In contrast, DL has GDR; that is, group delay varies with frequency, resulting in signal distortion and thus BER. To elaborate on the effects, the conventional Gaussian approximation given in [6] is rewritten, with GDR considered here.

Suppose the transfer function of DL has a unit amplitude and non-linear phase, given by

$$H_{DL}(\omega) = e^{j\Phi_{DL}(\omega)} \tag{6}$$

where $\Phi_{DL}(\omega) = -\omega T_d + \phi_\Delta(\omega)$, in which $\phi_\Delta(\omega)$ denotes the given attenuation component. The corresponding time-domain waveform is

$$\begin{aligned} h_{DL}(t) &= \mathscr{F}^{-1}\left[e^{j\Phi_{DL}(\omega)}\right] = \mathscr{F}^{-1}\left[e^{-j\omega T_d} \cdot e^{j\phi_\Delta(\omega)}\right] \\ &= \delta(t - T_d) * h_\Delta(t) \end{aligned} \tag{7}$$

where $h_\Delta(t)$ is the time-domain impulse response of $e^{j\phi_\Delta(\omega)}$. The output of DL, $\tilde{r}_i^{AT}(t - T_d)$, means the attenuation of $\tilde{r}_i(t - T_d)$, which can be calculated as follows:

$$\begin{aligned} \tilde{r}_i^{AT}(t - T_d) &= \tilde{r}_i(t) * h_{DL}(t) \\ &= \tilde{r}_i(t) * h_\Delta(t) * \delta(t - T_d) \\ &= \sum_{j=0}^{N_p/2-1} \sum_{l=1}^{L} [g_{AT}(t - jT_f - c_j T_c - T_d) \\ &\quad + b_i g_{AT}(t - jT_f - c_j T_c - 2T_d)] + \tilde{n}_{AT}(t - T_d) \end{aligned} \tag{8}$$

where $g_{AT}(t)$ and $\tilde{n}_{AT}(t)$ are the attenuations of $g(t)$ and $\tilde{n}(t)$, respectively, when passing DL, and can be obtained via

$$g_{AT}(t) = g(t) * h_\Delta(t) \tag{9}$$

and

$$\tilde{n}_{AT}(t) = \tilde{n}(t) * h_\Delta(t) \tag{10}$$

To this end, one obtains the decision statistic required for data detection as

$$Z_i = \sum_{j=0}^{N_P/2-1} \int_{jT_f + c_j T_c + T_d}^{jT_f + c_j T_c + T_d + T_I} \tilde{r}_i(t) \tilde{r}_i^{AT}(t - T_d)\, dt \tag{11}$$

Since b_j is equally probable, without loss of generality one may assume $b_j = +1$ in the following analysis.

Because the channel is assumed to be time-invariant over one frame, we note that

$$g\left(t - jT_f - c_j T_c\right) = g\left(t - jT_f - c_j T_c - T_d\right) \tag{12}$$

and

$$g_{AT}\left(t - jT_f - c_j T_c - T_d\right) = g_{AT}\left(t - jT_f - c_j T_c - 2T_d\right) \tag{13}$$

for all $t \in \left(0, T_I\right)$ and c_j. Thus, over the interval $t \in \left(0, T_I\right)$, we can employ $w(t) = g\left(t - jT_f - c_j T_c - T_d\right)$, $w_{AT}\left(t\right) = g_{AT}(t - jT_f)$, and $v_{d,j}\left(t\right) = \tilde{n}\left(t\right)$, $v_{r,j}^{AT}\left(t\right) = \tilde{n}_{AT}\left(t\right)$. So, (11) can be further simplified as

$$\begin{aligned} Z_i &= \sum_{j=0}^{N_p/2-1} \int_0^{T_I} \left(w(t) + v_{d,j}\left(t\right)\right)\left(w_{AT}\left(t\right) + v_{r,j}^{AT}\left(t\right)\right) dt \\ &= Z_{i,1} + Z_{i,2} + Z_{i,3} + Z_{i,4} \end{aligned} \tag{14}$$

where

$$Z_{i,1} = \sum_{j=0}^{N_p/2-1} \int_0^{T_I} w\left(t\right) w_{AT}\left(t\right) dt \tag{15}$$

$$Z_{i,2} = \sum_{j=0}^{N_p/2-1} \int_0^{T_I} w_{AT}\left(t\right) v_{d,j}\left(t\right) dt \tag{16}$$

$$Z_{i,3} = \sum_{j=0}^{N_p/2-1} \int_0^{T_I} w\left(t\right) v_{r,j}^{AT}\left(t\right) dt \tag{17}$$

$$Z_{i,4} = \sum_{j=0}^{N_p/2-1} \int_0^{T_I} v_{d,j}\left(t\right) v_{r,j}^{AT}\left(t\right) dt \tag{18}$$

Note that in (14), $Z_{i,1}$ is the desired signal, while $Z_{i,2}$, $Z_{i,3}$, $Z_{i,4}$ are noise terms. Under the action of GDR, the pulses and noise are distorted while passing non-ideal delay line in the receiver. Generation of $v_{r,j}^{AT}\left(t\right)\left(j = 0,1,...,N_p/2-1\right)$

and $w_{AT}(t)$ cause the difference between the $Z_{i,j}\ (j=1,2,3,4)$ here and the corresponding variables in [6].

Next, for convenient analysis, define

$$E_w = \int_0^{T_I} w^2(t)\,dt \tag{19}$$

$$E_{w,w_{AT}} = \int_0^{T_I} w(t)\,w_{AT}(t)\,dt \tag{20}$$

$$E_{w_{AT}} = \int_0^{T_I} w_{AT}^2(t)\,dt \tag{21}$$

Gaussian approximation (GA) analysis assumes that $Z_{i,j}\ (j=2,3,4)$ are mutually uncorrelated Gaussian random variables. When the time-bandwidth product is large enough, the assumption is valid due to the central limit theorem [1], [5]. So, the conditional mean of Z_i is given by

$$\begin{aligned} & E\{Z_i \mid \alpha_1,\alpha_2,\ldots,\alpha_{L_{CAP}}, d_0=+1\} \\ & = Z_{i,1} \\ & = \frac{N_p}{2} \times \sum_{l=1}^{L_{CAP}} \alpha_l^2 \times E_{w,w_{AT}} \end{aligned} \tag{22}$$

And the variance of $Z_{i,2}$ is obtained as

$$\begin{aligned} & \operatorname{var}\{Z_{i,2} \mid \alpha_1,\alpha_2,\ldots,\alpha_{L_{CAP}}, d_0=+1\} = \sum_{j=0}^{N_p/2-1} \sum_{j'=0}^{N_p/2-1} [\sum_{l=1}^{L_{CAP}} \alpha_l^2 \times \\ & \int_0^{T_I}\int_0^{T_I} w_{AT}(t)\,w_{AT}(u)\,R_{\tilde{n}}\left(t-u+(j-j')\times T_d\right) dt du] \\ & \approx \frac{N_p}{4} \times N_0 \times \sum_{l=1}^{L_{CAP}} \alpha_l^2 \times E_{w_{AT}} \end{aligned} \tag{23}$$

In (23), GA is obtained following [18].

It should be remarked that the spectral density of $\tilde{n}_{AT}(t)$ is equal to that of $\tilde{n}(t)$, because

$$
\begin{aligned}
S_{\tilde{n}_{AT}}(\omega) &= \left|H_{DL}(\omega)\right|^2 \times S_{\tilde{n}}(\omega) \\
&= S_{\tilde{n}}(\omega) \\
&= \frac{N_0}{2}
\end{aligned} \tag{24}
$$

So, $R_{\tilde{n}_{AT}}(\tau) = R_{\tilde{n}}(\tau)$. As a result, the variance of $Z_{i,3}$ as well as $Z_{i,2}$ can be computed according to

$$
\begin{aligned}
&\operatorname{var}\{Z_{i,3} \mid \alpha_1, \alpha_2, ..., \alpha_{L_{CAP}}, d_0 = +1\} = \sum_{j=0}^{N_p/2-1} \sum_{j'=0}^{N_p/2-1} [\sum_{l=1}^{L_{CAP}} \alpha_l^2 \times \\
&\int_0^{T_I}\int_0^{T_I} w(t)w(u) R_{\tilde{n}_{AT}}\left(t-u+(j-j')\times T_d\right) dtdu] \\
&\approx \frac{N_p}{4} \times N_0 \times \sum_{l=1}^{L_{CAP}} \alpha_l^2 \times E_w
\end{aligned} \tag{25}
$$

and the variance of $Z_{i,4}$ can also be derived as follows [5]:

$$
\operatorname{var}\{Z_{i,4} \mid \alpha_1, \alpha_2, ..., \alpha_{L_{CAP}}, d_0 = +1\} \approx \frac{N_p}{4} \times N_0^2 \times W \times T_I \tag{26}
$$

Thus, the conditional BER for TR-UWB systems with GDR can be found by evaluating

$$
\begin{aligned}
&P\{e \mid \gamma_w, \gamma_{w_{AT}}, \gamma_{w,w_{AT}}\} \\
&= Q\left(\frac{E\{Z_i \mid \alpha_1, \alpha_2, ..., \alpha_{L_{CAP}}, d_0 = +1\}}{\sqrt{\sum_{k=2}^{4} \operatorname{var}\{Z_{i,k} \mid \alpha_1, \alpha_2, ..., \alpha_{L_{CAP}}, d_0 = +1\}}} \right)
\end{aligned} \tag{27}
$$

Using (22), (23), (25) and (26), the expression (27) can be rewritten as

$$
\begin{aligned}
&P\{e \mid \gamma_w, \gamma_{w_{AT}}, \gamma_{w,w_{AT}}\} \\
&= Q\left(\frac{\gamma_{w,w_{AT}}}{\sqrt{\frac{\gamma_{w_{AT}}}{2} + \frac{\gamma_w}{2} + \frac{N_p}{4} \times W \times T_I}} \right)
\end{aligned} \tag{28}
$$

where

$$\gamma_w = \frac{N_p}{2N_0} \times \sum_{l=1}^{L_{CAP}} \alpha_l^2 \times E_w \tag{29}$$

$$\gamma_{w_{AT}} = \frac{N_p}{2N_0} \times \sum_{l=1}^{L_{CAP}} \alpha_l^2 \times E_{w_{AT}} \tag{30}$$

$$\gamma_{w,w_{AT}} = \frac{N_p}{2N_0} \times \sum_{l=1}^{L_{CAP}} \alpha_l^2 \times E_{w,w_{AT}} \tag{31}$$

Rewrite equation (12) in [6] as follows:

$$P\left\{e \middle| \gamma_{TR}\right\} = Q\left(\frac{\gamma_{TR}}{\sqrt{\gamma_{TR} + \frac{N_s}{4} WT}} \right) \tag{32}$$

where

$$\gamma_{TR} \triangleq \frac{E_s}{2N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2 \tag{33}$$

In (32) and (33), E_s is the energy per bit and N_s and T correspond to N_p and T_I, respectively. Comparing (28) to (32), γ_{TR} splits into γ_w, $\gamma_{w_{AT}}$ and $\gamma_{w,w_{AT}}$ due to GDR.

In (28), the only random variable is $\sum_{l=1}^{L_{CAP}} \alpha_l^2$ since $E_w, E_{w_{AT}}, E_{w,w_{AT}}$ are all constants for a given GDR function. Consequently, BER can be obtained by averaging the conditional BER, as

$$P_{e,GDR} = \int_0^\infty P\{e \mid x\} f_u(x) dx, \quad u = \sum_{l=1}^{L_{CAP}} \alpha_l^2 \tag{34}$$

where $f_u(x)$ denotes the general probability density function of $\sum_{l=1}^{L_{CAP}} \alpha_l^2$, and (34) can be computed by using any appropriate numerical method.

2.2 Effect of GDRR

GDR range (GDRR) is one of the parameters to characterize phase distortion. GDRR is defined as the peak-to-peak amplitude of GDR; thus, $GDRR = \delta_0$ means the group delay alternates between $T_d + \delta_0$ and $T_d - \delta_0$, where T_d is the expected delay. It also implies that the phase varies in the range of $\Phi(\omega) = (-T_d \pm \delta_0)\omega$.

In this subsection, we analyze the impact of GDRR on DL with respect to the degradation of BER.

A given GDRR gives an infinite set of GDR, corresponding to an infinite number of DL transfer functions. There is a one-to-one correspondence between $H_{DL}(\omega)$ and $g_{AT}(t)$. So $E_w, E_{w_{AT}}, E_{w,w_{AT}}$ become random variables but they are independent against $\sum_{l=1}^{L_{CAP}} \alpha_l^2$. As a result, (34) needs to be modified as

$$P_{e,GDRR} = E\left[\int_0^\infty P\{e \mid x, H_{DL}(w)\} f_u(x)dx\right], \quad u = \sum_{l=1}^{L_{CAP}} \alpha_l^2 \tag{35}$$

The evaluation of $P_{e,GDRR}$ can be obtained by a Monte Carlo algorithm, as shown below.

3 Numerical Results and Analysis

In this section, we make a comparison between theoretical result and simulation result.

The signal pulse considered here is a Gaussian pulse, which has a 2*ns* pulse width and modulated by multiplying a 3.5GHz cosine wave, as follows:

$$p(t) = e^{-\frac{1}{2}\times\left(\frac{t}{0.707}\right)^2} \cos\left(2\pi \times 3.5 \times 10^9 \times t\right) \tag{36}$$

The pulse has about 1GHz bandwidth with $-10\,\text{dB}$ power decrease. The system is restricted to an independent Nakagami-*m* fading channel with a uniform power delay profile (PDP), in which $L_{CAP} = 5$ and the PDF of $\alpha_l\left(1,2,\ldots,L_{CAP}\right)$ is

$$f\left(\alpha_l;m_l,\Omega_l\right)=\frac{2}{\Gamma\left(m_l\right)}\left(\frac{m_l}{\Omega_l}\right)^{m_l}\alpha_l^{2m_l-1}\exp\left(-\frac{m_l}{\Omega_l}\alpha_l^2\right) \tag{37}$$

where $\Omega_l = E\{\alpha_l^2\}$ is the mean-square value of the amplitude, $\Gamma\left(\cdot\right)$ is the Gamma function. and

$$m_i = \Omega_i^2 \big/ Var\{\alpha_l^2\} \geq 1/2 \tag{38}$$

is the fading parameter.

Thus, $\sum_{l=1}^{L_{CAP}}\alpha_l^2$ obeys a gamma distribution, with PDF [19]

$$f\left(u;k,\theta\right)=u^{k-1}\frac{e^{-u/\theta}}{\theta^k\Gamma\left(k\right)} \tag{39}$$

for

$$u=\sum_{l=1}^{L_{CAP}}\alpha_l^2, k=\sum_i m_i, \theta=\sum_i\Omega_i \Big/ \sum_i m_i$$

So, $f_u(x)$ should be substituted by $f\left(u;k,\theta\right)$ in (34).

It should be noted that the severity, depth, and amplitude of Nakagami channel model are controlled by m_i [20]. For example, it results in wide-spread Rayleigh-fading model when $m_l = 1$ and one-sided Gaussian distribution when $m_l = 0.5$. In addition, if $m_l > 1$, Nakagami-*m* distribution closely approximates Rice distribution; if $0.5 < m_l < 1$, it fades more severely than the Rayleigh fading, but this case is close to the typical UWB channels. So, we let $m_l = 0.8$ in our simulation below for simplicity.

We set $\Omega_l = 1/L_{CAP}\ (l = 1,2,...,L_{CAP})$ due to the restriction of $E\left[\sum_{l=1}^{L_{CAP}} \alpha_l^2\right] = 1$.

The other parameters are set as follows: $W = 10GHz$, $N_f = 1$, $T_I = 10ns$, $T_f = 40ns$, $T_d = 20ns$.

3.1 BER Performance Corresponding to a Given GDR

The transfer function of DL is assumed to be

$$H_{DL}(\omega) = \exp\left(j\Phi(\omega)\right), \Phi(\omega) = \sum_{i=0}^{6} PI_i \times \omega^{6-i} \tag{40}$$

where

$$\begin{aligned} PI_0 &= -8.715357270684494 \times 10^{-62} \\ PI_1 &= 2.172219023681759 \times 10^{-51} \\ PI_2 &= 2.168669655160014 \times 10^{-40} \\ PI_3 &= -9.416205209334618 \times 10^{-30} \\ PI_4 &= 1.149815106311333 \times 10^{-19} \\ PI_5 &= -3.953449184128336 \times 10^{-10} \\ PI_6 &= 0.102247186947813 \end{aligned} \tag{41}$$

DL has a uniform amplitude-frequency characteristic. In addition, $\Phi(\omega)$ is described by a polynomial of degree 6. The reason is as follows. According to [21], [22], a group delay of the real delay line ripples within a range around the expected delay, randomly and non-periodically. The ripple is named GDR and its range is named GDRR here. A non-periodical curve of function randomly generated within this range can be considered as a GDR close to reality. Meanwhile, the corresponding phase varies within $-(T_d \pm GDRR) \cdot \omega$.

Curve fitting is used to obtain the required phase by fitting a curve to the sample points which are first randomly generated among the given GDRR. The GDRR can be set to any value. Here, we assume GDRR=0.1ns. Among the existing methods of curve fitting, it is found that a "cubic spline" curve can accurately pass the sample points and shows satisfactory characteristics of the randomly ripple and non-periodicity. “Polynomial” comes next, which also meets the requirement, but has less distinct fluctuations with a simpler expression than “cubic spline”. “Sum of Sin Functions” and “Fourier” will introduce periodic components. “Exponential” and “Gaussian” are not as well in constructing the expected ripple. The results of “Rational” are easily going beyond the restricted boundary. “Weibull” and “Power” cannot be fit to non-positive sample data. This paper focuses on certifying the accuracy and efficiency of the proposed method. When the phases meet the requirements, different forms of their expressions will provide the same results. So, “Polynomial” is the best choice for the purposes of the paper.

The degree of polynomial is an experienced value which should be chosen among 5 to 7. If the value is too large, the phase will go beyond the restricted boundary; if too low, the fluctuation of the phase will be non-distinct. So, the degree is chosen to be 6 here. To that end, the scheme will randomly generate a phase subject to the given GDRR. Thus, $\Phi(\omega)$ with coefficient (41) is obtained.

Group delay can also be simply calculated, as follows:

$$\tau(\omega) = -\frac{d\Phi(\omega)}{d\omega} = -\sum_{i=0}^{5}(6-i)PI_i \times \omega^{5-i} \tag{42}$$

Phase-frequency characteristic without mean value is shown in Fig.3 and the corresponded GDR is shown in Fig.4.

The original Gaussian pulse and the distorted one, which passes the DL, are shown in Fig.5. In the time domain, pulse is dispersed when DL possesses GDR.

In Fig.6, the solid line with circles is the reference curve, which shows the BER performance of TR-UWB system with an ideal delay line. The curves with asterisks represent the impact of the assumed DL. It can be seen that it is serious enough to cause BER degradation relative to the curve with an ideal DL, although the group delay varies within a tiny range of 0.1ns, and the pulse spreads insignificantly in(40). Furthermore, this simulation result shows a good agreement with the theoretical result.

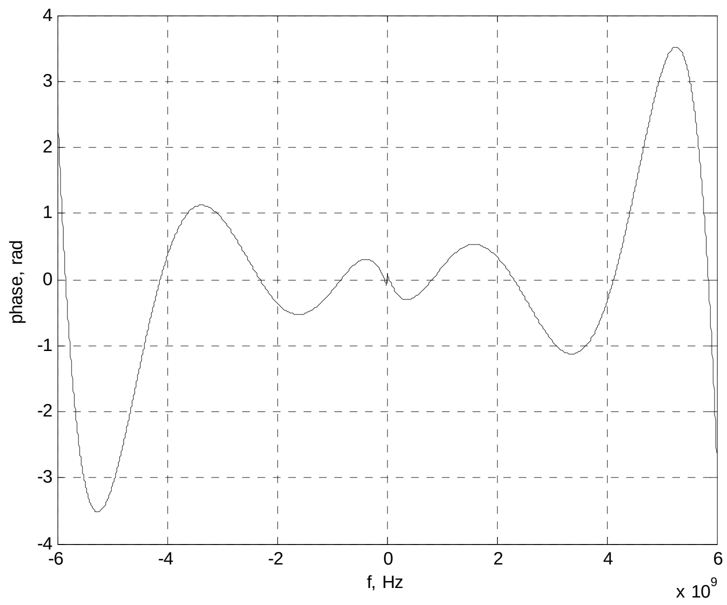

Figure 3 Delay line phase-frequency characteristic without a mean value

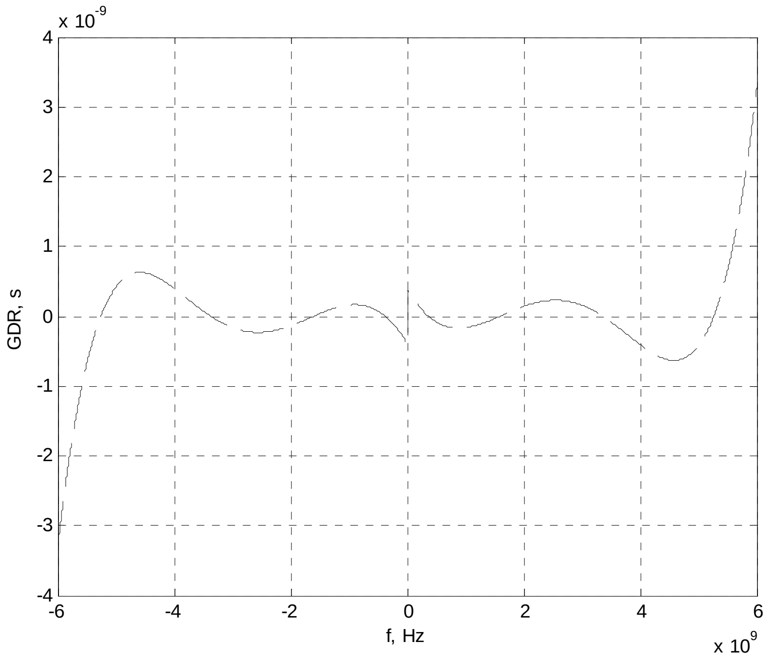

Figure 4 GDR corresponding to the delay line phase-frequency characteristic without a mean value, which is shown in Fig.3

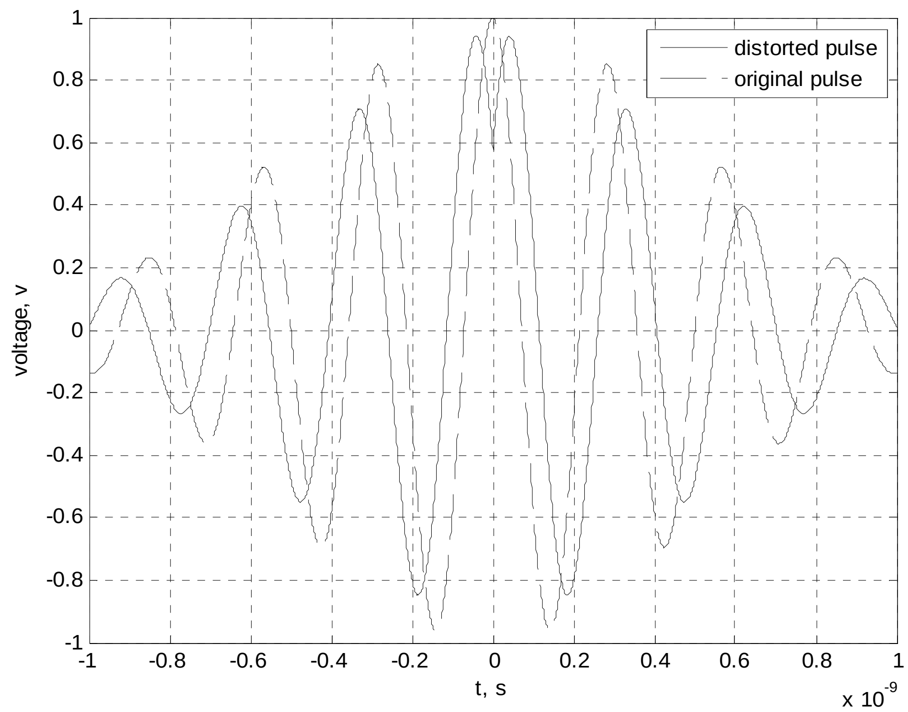

Figure 5 Original Gaussian pulse and distorted Gaussian pulse

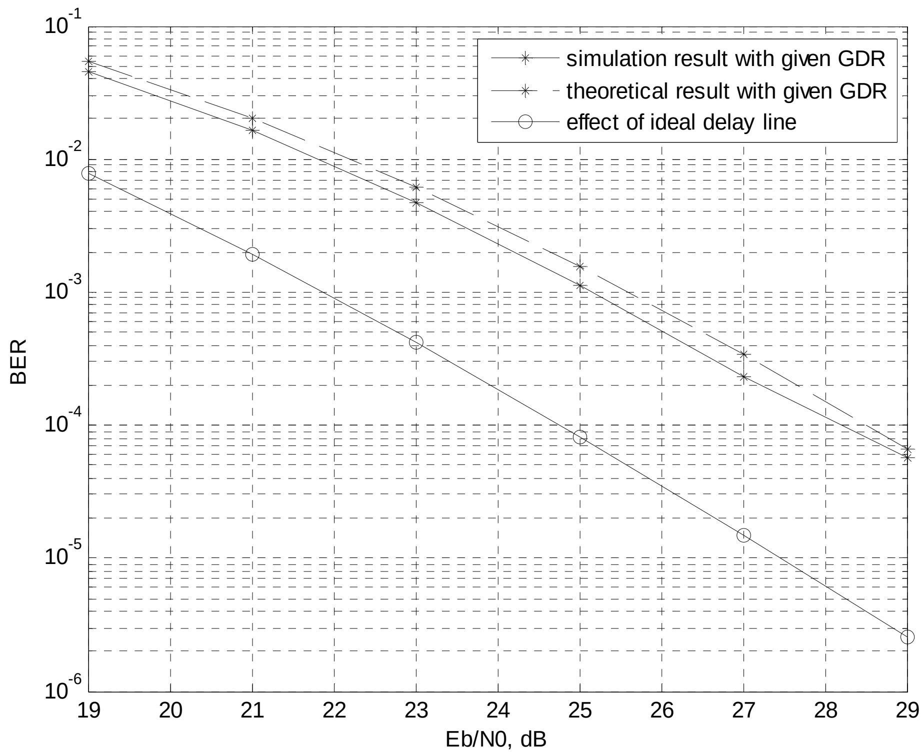

Figure 6 Comparison between the BER performance of TR-UWB systems with a given GDR and the one with an ideal DL

3.2 BER Performance Corresponding to a Given GDRR

When designing a DL, one usually would like to determine the acceptable variation of the group delay. A crucial parameter is GDRR, discussed above. Larger GDRR implies that GDR varies more significantly and BER degrades more severely. So, smaller GDRR means better BER performance in general. In this case, DL is near perfect but more complicated to implement for the need of higher-order circuits.

In this subsection, we will evaluate the impact of GDRR on BER, denoted by $P_{e,GDRR}$, to trade off the system performance and the circuitry complexity in designing DL.

There are infinite DLs obeying the same GDRR, while a DL corresponds to a $P_{e,GDR}$, so $P_{e,GDRR}$ should be the statistical average of $P_{e,GDR}$. Randomly regenerate a DL transfer function which obeys the same GDRR for 10^6 times in simulations. The probability density distribution (PDD) of $P_{e,GDR}$ is regarded as a normal distribution by the central limit theorem. It can be easily fitted by a Gaussian function, as illustrated by Fig.7.

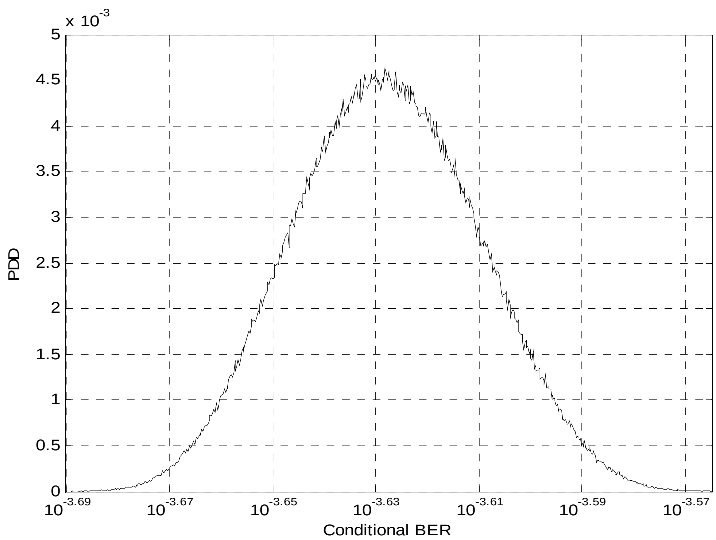

Figure 7 PDD of $P_{e,GDR}$ when $E_b/N_0 = 24dB$ and $GDRR = 0.02ns$

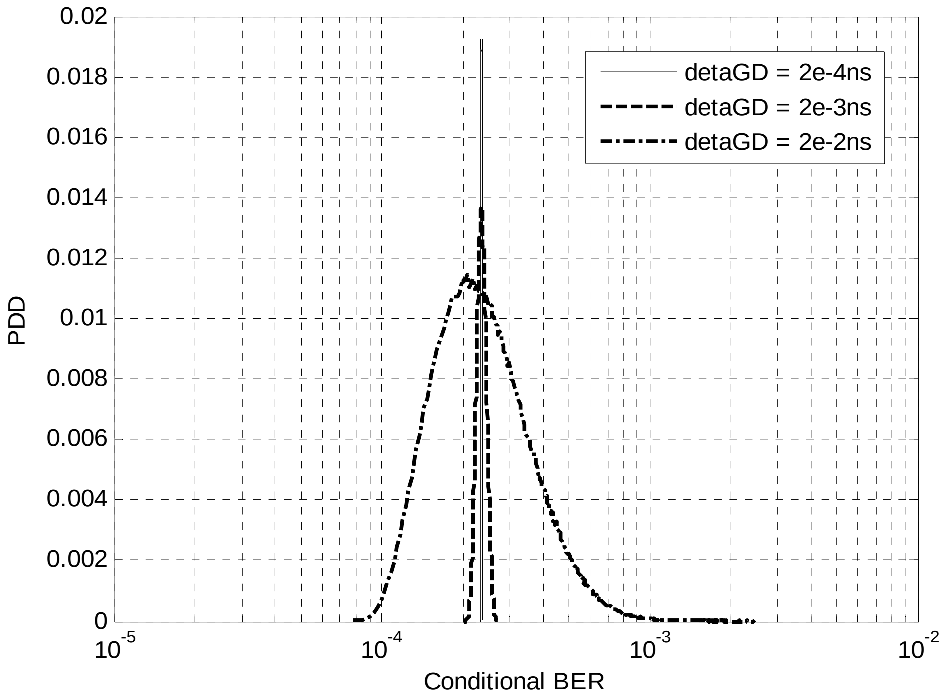

Figure 8 PDD of $P_{e,GDR}$ when $E_b/N_0 = 24dB$ and GDRR is equal to, respectively, $2\times10^{-4}ns, 2\times10^{-3}ns, 2\times10^{-2}ns$.

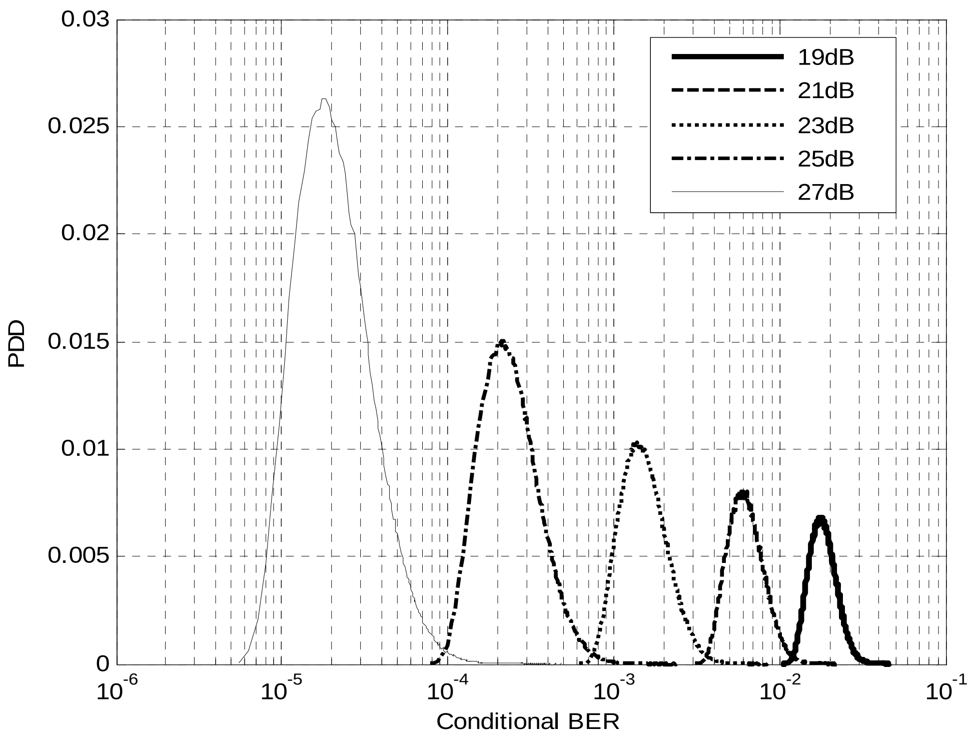

Figure 9 PDD of $P_{e,GDR}$ when $GDRR = 0.02ns$ and E_b / N_0 is equal to, respectively, 19dB, 21dB, 23dB, 25dB, 27dB

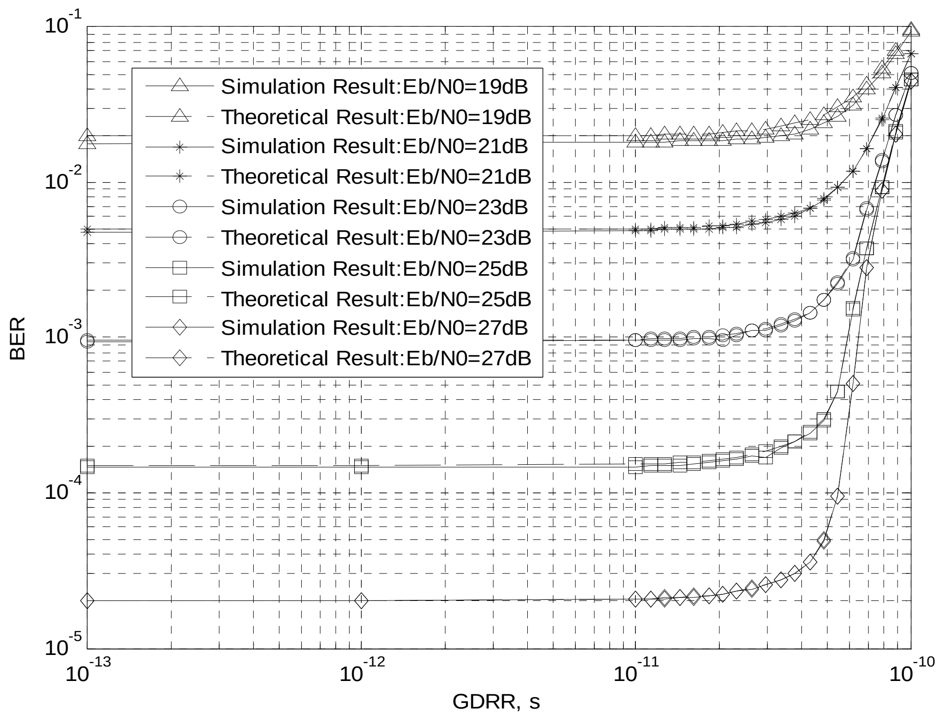

Figure 10 Comparison between simulation result and theoretical result corresponding to different GDRR from $1\times10^{-4}ns$ to $1\times10^{-1}ns$ when E_b/N_0 is equal to, respectively, 19dB, 21dB, 23dB, 25dB, 27dB

Fig.8 and Fig.9 show that different E_b/N_0 or different GDRR will both result in different PDDs. So, we have to use numerical method to obtain the $P_{e,GDRR}$.

Fig.10 depicts the effect of GDRR on TR-UWB system with a Nakagami-*m* fading channel, under the assumption that the power delay profile (PDP) is uniform and $m_l = 0.8$. GDRR is tested from 0.0001ns to 0.1ns. The dashed lines are the theoretical results according to (35), where the simulation results are given by the solid lines, which show a close agreement with the theoretical results.

In Fig.10, notice that when GDRR is less than 0.01ns, BER is nearly constant. This domain is referred to as the flat BER domain. In this domain, smaller GDRR means that DL should be designed using more complex circuitry, which however yields little improvement of BER. Here, the optimized GDRR is 0.01ns. As GDRR increases from 0.01ns to 0.1ns, BER is degraded since the transmitted pulses have been distorted. This domain can be referred to as the degraded BER domain. In spite of this, one can choose the value according to practical considerations, such as a fabrication process or a tolerant BER performance requirement, so as to achieve a trade-off between the system performance and the design complexity. For example, if $BER = 5\times10^{-5}$ is tolerant in a particular system when $E_b / N_0 = 27dB$, GDRR should be just restricted within 0.05*ns* according to Fig.10.

It should be noted that this method has some limitations. First, the quasi-analytical/simulation approach [4] is used, because evaluation of (34) is difficult as pointed out in [1], [2], [4], [6]. For this reason, a simple relation between BER and GDR is calculated numerically, giving of a non-closed form solution. Second, $P_{e,GDRR}$ is only a value of expectation, so the designed DL needs to be further testified.

Nevertheless, the advantages of the new approach are quite obvious. Optimal GDRR is valuable for designing DL because it provides a design precision. Formula (35) suggests an efficient design method for determining the optimized GDRR. According to Fig. 10, a designer can easily get an appropriate GDRR. Besides, the performance of the designed DL can be conveniently testified using (34). As a result, it improves the traditional empirical design method of DL. When the time-bandwidth product is large, GA is an effective approach for analyzing the performance of TR-UWB systems. In fact, GA outperforms KL in this case although its solution is not in a closed form.

4 Conclusions

In this paper, differing from the traditional empirical design method, we have proposed a semi-analytic approach to designing a delay line for a TR-UWB system with an autocorrelation receiver. To accomplish the task, we have analyzed the BER degradation caused by the group delay ripple (GDR) range on the system in independent Nakagami-*m* fading channels. In so doing, we have derived explicit expressions for the conditional BER resulted from GDR, and have analyzed the GDR effects on BER.

We have found that GDRR-BER theoretical curves can be divided into a flat BER domain and a degraded BER domain, which can help decide an apt phase distortion when designing a DL. In the flat BER domain, more complexity of DL results in smaller GDRR but has little improvement on BER. Thus, relatively large GDRR may be chosen as a design index. In the degraded BER domain, BER performance is rather sensitive to GDRR, in the sense that lager GDRR means simpler DL configuration but worse BER performance. So, one has to choose a proper GDRR in practical design to achieve the trade-off between system performance and design complexity.

To summarize, although the solution is not in a closed form and the designed DL needs to be further testified, the new method developed in this paper seems to be practical in determining an acceptable phase distortion or assessing delay line performance of transmitted-reference ultra-wideband systems.

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