Circuits Syst Signal Process
DOI 10.1007/s00034-008-9067-2

NONLINEAR CIRCUITS AND SYSTEMS

Promising Performance of PA-Coded SIMO FM-DCSK Communication Systems

Chaoxian Zhang · Lin Wang · Guanrong Chen

Received: 14 December 2007 / Revised: 24 February 2008

Abstract Frequency-modulated differential chaos shift keying (FM-DCSK) is an attractive scheme which combines chaotic modulation with the spread-spectrum property. It is simple to implement and robust in multipath fading channels. A single-input multiple-output (SIMO) FM-DCSK architecture has recently been developed to increase data rate and achieve diversity gain. In this paper, several main channel coding schemes, i.e., error-correcting coding schemes, are introduced into the SIMO FM-DCSK communication system, with different code rates and different frame lengths, over multipath fading channels. It is found that, in contrast to low density parity check codes and convolutional codes, product accumulate (PA) codes can provide outstanding bit error rate performance improvement to the existing SIMO FM-DCSK system. In this paper, moreover, the optimum code rate for the PA-coded SIMO FM-DCSK system is investigated through simulations at a medium frame length. Because of its merits in several aspects, such as prominent advantages in better performance, simple encoding and decoding structures, and a flexibly adjustable code rate, this paper demonstrates that PA code is a strong candidate as the error-correcting scheme for

This research was partially supported by the City University of Hong Kong under the SRG Grant 7002274, the Dept. of Education Administration of China (NCET-04-0601), NSFC (No. 60272005), as well as the Key Project of Science and Technology in Fujian Province of China (No. 2006H0039).

C. Zhang · L. Wang (✉)
Dept. of Communication Engineering, Xiamen University, Fujian 361005, People's Republic of China
e-mail: wanglin@xmu.edu.cn

C. Zhang
e-mail: chaoxianzh@gmail.com

G. Chen
Dept. of Electronic Engineering, City University of Hong Kong, Hong Kong SAR, People's Republic of China
e-mail: eegchen@cityu.edu.hk

BIRKHÄUSER

the SIMO FM-DCSK system in transmitting medium and long frame lengths over multipath fading channels.

Keywords FM-DCSK · SIMO communication system · Error-correcting code · Error performance

1 Introduction

Chaos-based communication theory applies a chaotic modulation scheme, which is different from the conventional modulation schemes in that a non-periodic chaotic signal is used as the carrier [10]. Among the several chaotic modulation schemes, frequency-modulated differential chaos shift keying (FM-DCSK) offers the best robustness in the multipath environment with channel imperfection [4]. With the proposal of the M-ary FM-DCSK [3], a recent study has revealed more insights on the essence and advantages of FM-DCSK [15]. In order to increase the data rate and to achieve diversity gain by employing higher-order Walsh functions at the transmitter end and using multiple antennas at the receiver end, a single-input multiple-output (SIMO) FM-DCSK architecture has been proposed and evaluated recently. It is found that the uncoded SIMO FM-DCSK scheme outperforms the DS-VBLAST (direct spreading Vertical Bell Laboratories Layered Space-Time architecture) at a bit error rate (BER) of 10^{-6} over multipath fading channels [11].

On the other hand, it is known that error-correcting codes usually can be applied to an uncoded modulation system against noise interference. Some error-correcting codes, such as low density parity check (LDPC) codes and woven convolutional codes (WCCs), have been introduced to FM-DCSK systems before, where a significant performance improvement provided by the coding gain is evident [12, 14].

In this paper, a coded SIMO FM-DCSK system is presented, and the adaptability between the error-correcting codes and the SIMO FM-DCSK system will be explored over multipath fading channels. The simulation results demonstrate that product accumulate (PA) codes can provide significant BER performance improvement to the SIMO FM-DCSK system in contrast to LDPC codes and convolutional codes at both medium and high code rates and with both medium and long frame lengths. To that end, the optimum code rate for the PA-coded SIMO FM-DCSK system will be investigated through systematic simulations at a fixed medium frame length.

The rest of the paper is organized as follows. Section 2 shows the configuration of the SIMO FM-DCSK system, followed by a brief review of the original FM-DCSK scheme, while Sect. 3 describes the principle of PA codes and Sect. 4 shows the simulation results through a BER performance comparison among PA, LDPC, and convolutional coded SIMO FM-DCSK systems. The conclusions are drawn in Sect. 5.

2 The SIMO FM-DCSK System

2.1 Original FM-DCSK Modulator and Demodulator

The FM-DCSK system uses a frequency-modulated chaotic signal as the carrier, with a DCSK modulator for transmission. A chaotic signal is generated by a suitable

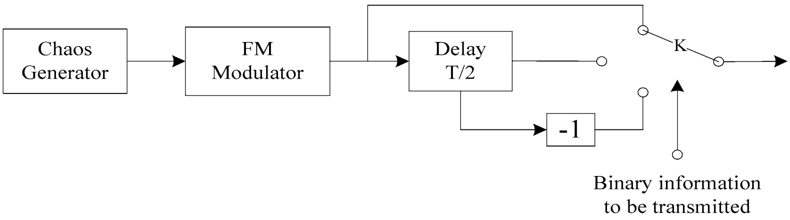


Fig. 1 Binary FM-DCSK modulator

chaotic map [10], and the simple cubic chaotic map is chosen here for implementation. As a binary FM-DCSK modulator, the whole setting is illustrated in Fig. 1.

The binary FM-DCSK modulation unit transmits a reference segment (or its repeated or reverse segment) of the frequency-modulated chaotic signal according to the digital information "1" or "0", respectively. The modulated signal is represented by two orthogonal basis functions, $g_1(t)$ and $g_2(t)$, as follows:

$$\begin{aligned} s_m(t) &= s_{m1}g_1(t) + s_{m2}g_2(t), \\ (s_{11} \quad s_{12}) &= \left(\sqrt{E_\text{b}} \quad 0\right), \\ (s_{21} \quad s_{22}) &= \left(0 \quad \sqrt{E_\text{b}}\right), \end{aligned} \tag{2.1}$$

where $s_m(t)$ is the modulated signal for transmission and E_b is the bit energy. The two orthogonal basis functions are

$$\begin{aligned} g_1(t) &= \begin{cases} +\frac{1}{\sqrt{E_\text{b}}}c(t), & 0 \le t < T/2, \\ +\frac{1}{\sqrt{E_\text{b}}}c(t-T/2), & T/2 \le t < T, \end{cases} \\ g_2(t) &= \begin{cases} +\frac{1}{\sqrt{E_\text{b}}}c(t), & 0 \le t < T/2, \\ -\frac{1}{\sqrt{E_\text{b}}}c(t-T/2), & T/2 \le t < T, \end{cases} \end{aligned} \tag{2.2}$$

where T is the bit duration and $c(t)$ is the frequency-modulated chaotic carrier, with the bit energy normalized to one. These forms of the orthogonal functions (2.2) are simply called "chips" for our discussion.

Here, the differential modulation process follows the second-order Walsh functions:

$$W_2 = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}. \tag{2.3}$$

The two row vectors w_1 and w_2 are multiplied to the carrier segment as the weights when the digital information is "1" or "0", respectively.

Due to its differential modulating characteristic, it is not necessary for the FM-DCSK unit to recover the carrier at the receiver end, and it does not require precise synchronization. During the demodulation process, the generalized maximum

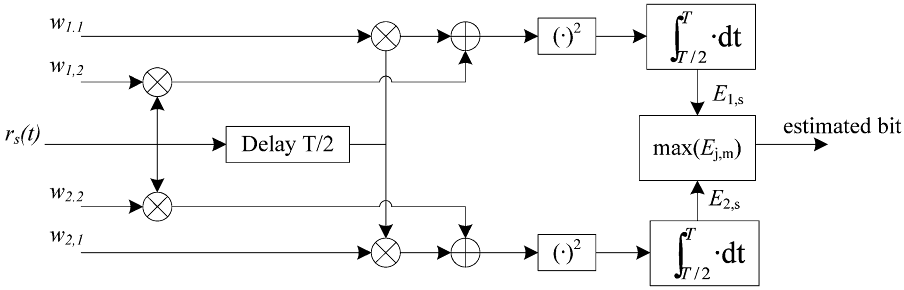


Fig. 2 FM-DCSK demodulator using GML detection

likelihood (GML) detection rule [3] as a universal method will be applied to the second-order Walsh functions used here and also to the higher-order Walsh functions employed by the transmitter. The FM-DCSK demodulator using GML detection is illustrated in Fig. 2.

As shown in Fig. 2, the weighted energy received is

$$E_{j,s} = \frac{1}{2} \int_{T/2}^{T} \left[r_s(t) \cdot w_{j,2} + r_s(t - T/2) \cdot w_{j,1} \right]^2 \mathrm{d}t, \quad j = 1, 2, \tag{2.4}$$

where T is the bit duration, $r_s(t)$ is the received signal, and $w_{j,i}$ $(i, j = 1, 2)$ are the corresponding elements in the second-order Walsh functions which are being multiplied as the weights during the modulation. The demodulator needs to find the index j that maximizes $E_{j,s}$ $(j = 1, 2)$ and then make a decision on the received bit, either “1” or “0”.

2.2 Configuration of SIMO FM-DCSK Transceiver

The above-described Walsh function-based FM-DCSK modulation scheme is now extended to multiple sub-streams transmitted with a single antenna to achieve a higher data rate by employing higher-order Walsh functions.

Consider a system with M sub-streams and N receiver antennas, simply denoted by (M, N). The M parallel sub-streams are modulated with the FM-DCSK method and then transmitted through a single antenna simultaneously, thus achieving a higher data rate that is M times that of the original FM-DCSK with the same spreading factor.

In order to serve the M sub-streams, $2M$-order Walsh functions are used for implementation. The length of each bit in any sub-stream will be divided into $2M$ segments, and each segment is weighted by its corresponding element chosen from the $2M$-order Walsh functions, as shown in Fig. 3.

Let β denote the chip length of each carrier segment and let f denote the global spread-spectrum factor. During the modulation, set $\beta = f/2M$ to ensure that the global spread-spectrum factor f is constant.

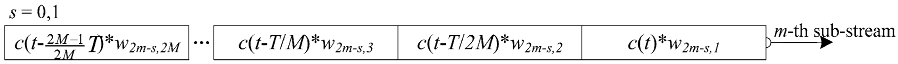


Fig. 3 Signal structure of m-th sub-stream

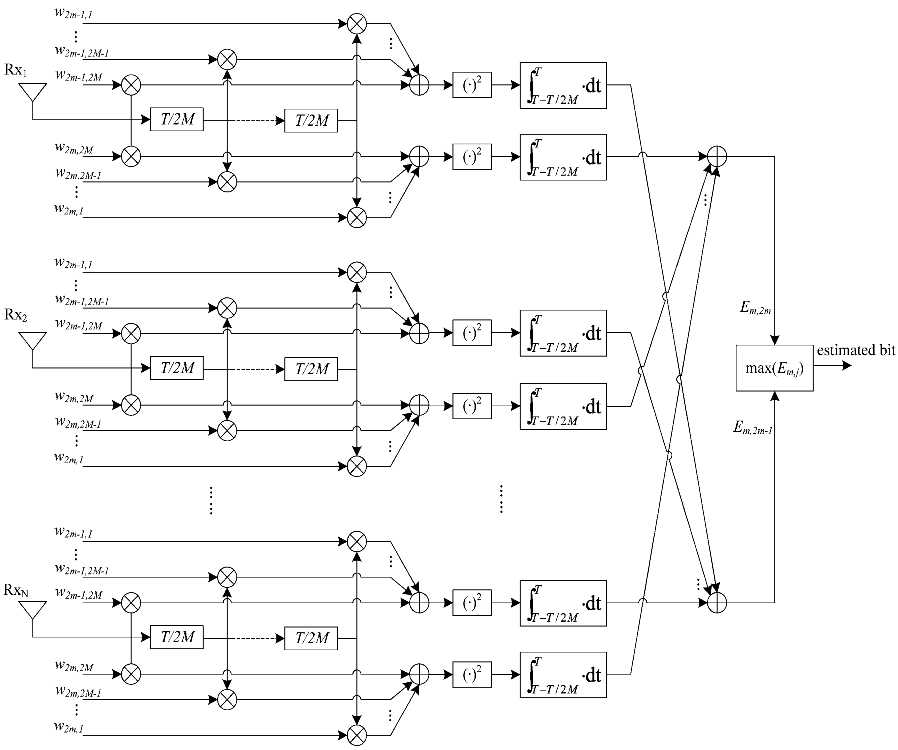


Fig. 4 Receiver architecture for detecting the m-th sub-stream

Over an L-tap multipath fading channel, the received signal in receiver antenna n at time t is

$$r_n(t) = \sum_{m=1}^{M} \sum_{l=1}^{L} h_n(l) x_m(t-l) + N_n(t), \tag{2.5}$$

where $h_n(l)$ is the l-th channel tap gain from the single transmit antenna to receiver antenna n, and $N_n(t)$ is the additive white Gaussian noise (AWGN) at time t received by antenna n, with zero mean and variance σ_n^2. The tap gains, $h_n(l)$, are modeled as independent and identically distributed (i.i.d.) Rayleigh-distributed random variables, with zero mean and variance 1, for each channel (after normalization).

The detection process is carried out independently through each receiver antenna, with the corresponding Walsh functions used in the respective sub-stream. Then, the weighted energy calculated from each receiver antenna is combined for final decision making, as illustrated in Fig. 4.

3 Basic Principle of PA Codes

3.1 Error-Correcting Codes

Channel coding, also called error-correcting coding, refers to the class of signal transformations designed to improve communication performance by enabling the transmitted signals to better withstand the effects of various channel impairments, such as noise, interference, and fading [9].

The basic principle and procedure are to insert structured redundancy into the source data so that the presence of errors can be detected or the error can be corrected. The encoding procedure provides the coded signals with better distance properties than those of their uncoded counterparts [9]. The ratio of data bits to coded bits is defined to be the code rate R_c ($0 < R_c \leq 1$). The smaller the R_c, the lower the bandwidth efficiency, but usually (not always) the higher the capability of error detection and correction.

3.2 PA Codes

PA codes, first proposed in 2001 [6], were shown to possess many desirable properties: close-to-capacity performance, low encoding and decoding complexity, regular structure, and flexible rate adaptation for all rates above $1/2$ [7]. On the other hand, LDPC codes [2, 8] have attracted a great deal of attention in the field of coding theory in recent years, because they can provide good performance at low decoding complexity. It is found that the best irregular LDPC code can achieve within 0.04 dB of the Shannon limit at a BER of 10^{-6}, but its frame length ($N = 10^7$) is too large for practical applications [1]. The performance comparison between PA codes and regular LDPC codes has been investigated, e.g., in [13], with simulation results demonstrating that the former outperforms the latter with long frame lengths at rates above $1/2$ over AWGN channels. It may be foreseen that PA codes are also a good candidate for future digital communication systems.

PA codes are a class of interleaved serial concatenated codes where the inner code is a rate-1 recursive convolutional code $1/(1 + D)$ (also known as the accumulator) and the outer code takes the form of two parallel branches of single-parity check (SPC) codes concatenated via a random interleaver (Fig. 5). In each branch, p blocks of frames from $(t + 1, t)$ SPC codes are combined and interleaved together. These codes have parameters $(N, K, R_c) = (p(t + 2), pt, t/(t + 2))$ with N, K, R_c representing the frame length, data block length, and code rate, respectively. It has been shown that under iterative decoding, PA codes that are linear time encodable and linear time decodable can provide a performance similar to that of turbo codes but with significantly lower decoding complexity with a lower error floor [7]. In addition, PA codes are simple to construct and have a very regular structure, which makes them easy to implement.

4 Simulation Results

On the basis of all the above, sufficient simulations have been carried out to evaluate the performance of a PA-coded SIMO FM-DCSK system in contrast to LDPC and

Fig. 5 Encoding model for PA codes

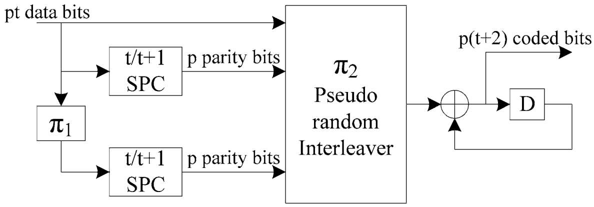
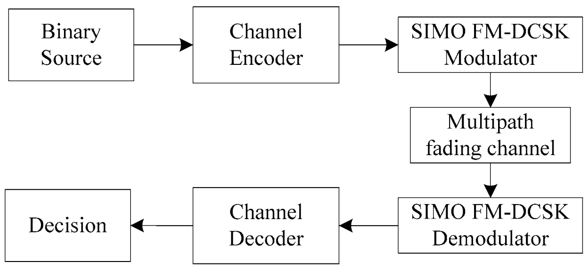


Fig. 6 Block diagram of coded SIMO FM-DCSK system



convolutional coded ones. The basic model of these coded SIMO FM-DCSK systems is illustrated in Fig. 6.

A simple cubic chaotic map is chosen here for implementation in the FM-DCSK modulator. Note that the channel coding scheme here includes three different kinds: LDPC, PA, and convolutional codes. Simulations were thus performed by applying these three codes over multipath fading channels. The simulation parameters include a chaotic number generator using a cubic map, global spread-spectrum factor $f = 64$ for FM-DCSK, transmitting sub-stream number $M = 2$, and receiver antenna number $N = 4$. The parameters of the multipath fading channel include a multipath number of 3, a power distribution of $\{0.4, 0.4, 0.2\}$, and delays of $\{0, T/f, 2T/f\}$. Moreover, the encoding and decoding methods for each type of code are chosen as follows: the irregular parity-check matrix H and the BP (believe propagation) decoding algorithm are employed for LDPC codes, the decoding scheme for PA is a min-sum algorithm, and the convolutional coding structure is (2, 1, 3) with the MAP (maximum a posteriori) decoding algorithm.

The BER performance of the uncoded SIMO FM-DCSK scheme with $f = 128$ and variables M and N has been investigated in [11], and the results are illustrated in Figs. 7 and 8. Figure 7 shows the performance of the uncoded SIMO FM-DCSK scheme, where there are M variable sub-streams and 2 fixed receiver antennas. In spite of the orthogonal characteristic of Walsh functions, increasing M leads to a somewhat worse performance. In Fig. 8, $(2, N)$-uncoded systems are simulated and the obtained results plotted. One can see that an increase in N improves the performance significantly, which signifies a diversity gain of the proposed system.

Figure 9 shows the BER performance of coded SIMO FM-DCSK systems with code rate $1/2$. One can see that no matter whether the system is PA, LDPC, or convolutional coded, the BER performance improves with the increase in the frame length from 1 200 to 9 600 bits. Meanwhile, PA and LDPC-coded systems both present a much better error-correcting ability than the convolutional coded system at a BER of 10^{-6}. Because of the short interleave length, one can see the error floor appear-

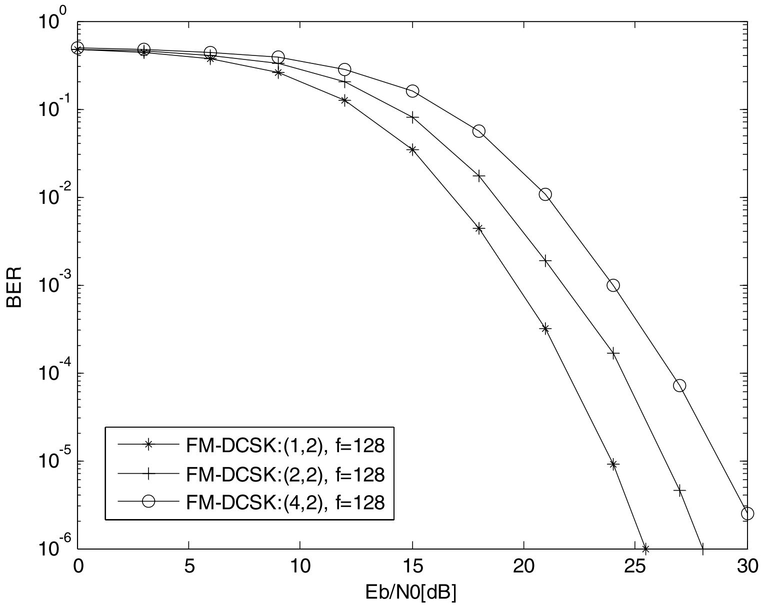


Fig. 7 Performance of the uncoded SIMO FM-DCSK scheme with M sub-streams and 2 receiver antennas over a multipath fading channel, with $M = 1, 2, 4$

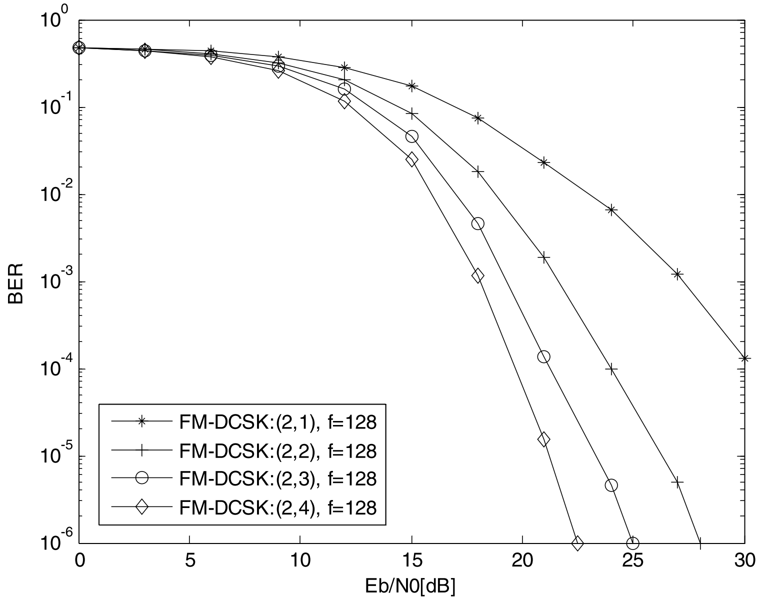


Fig. 8 Performance of the uncoded SIMO FM-DCSK scheme with 2 sub-streams and N receiver antennas over a multipath fading channel, with $N = 1, 2, 3, 4$

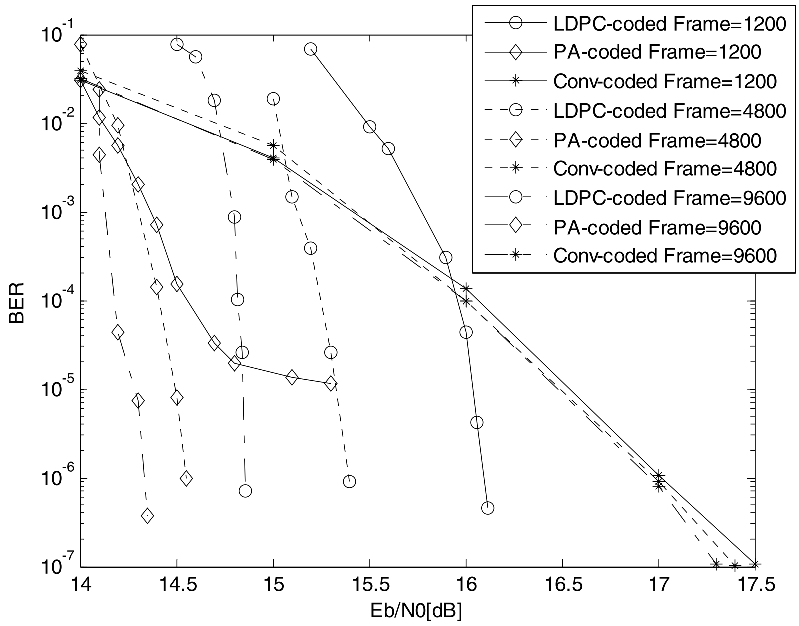


Fig. 9 BER performance of coded SIMO FM-DCSK systems with code rate 1/2 over multipath fading channels

ing in the PA-coded system with a frame length of 1 200 bits. The error floor can also be seen from the figures reported in [5], which demonstrates the performance of the PA codes over a fading channel. Although the PA-coded system has an error floor under the condition of a short frame length, it achieves an outstanding performance improvement over that of the LDPC-coded systems with the increase of their corresponding frame lengths. Through the comparison between PA-coded and LDPC-coded systems, with code rate $2/3$ as shown in Fig. 10, it can be seen that the observation described above remains the same.

Moreover, it can be seen from Figs. 9 and 10 that with the increase of the code rate from $1/2$ to $2/3$, the performance of the PA-coded SIMO FM-DCSK system is even better. More precisely, the position of the error floor in a PA-coded system at code rate $2/3$ is a little lower than that of the ones at code rate $1/2$ with 1 200-bit frame length; at the same time, it is found that the former performance curve can save at least 0.25 dB more than the latter at a BER level of 10^{-5}. It can be observed that the PA-coded systems at the $2/3$-code rate with frame lengths of 4 800 bits and 9 600 bits have a power savings of about 0.2 dB and 0.1 dB at a BER of 10^{-6} compared with the $1/2$-code rate systems, respectively. The performance gains of the PA-coded systems compared to the LDPC-coded systems are also increasing from 9 600-bit to 4 800-bit frame lengths in Figs. 9 and 10, no matter which code rate is adopted, $1/2$ or $2/3$; at the same time, this advantage is more obvious when the code rate is $2/3$ in Fig. 10. This means that there may be an optimum code rate for these systems. The performance curves of the minimum required E_b/N_0 vs. different code rates for both reliable voice transmission (BER $= 10^{-3}$) and data transmission (BER $= 10^{-6}$) in

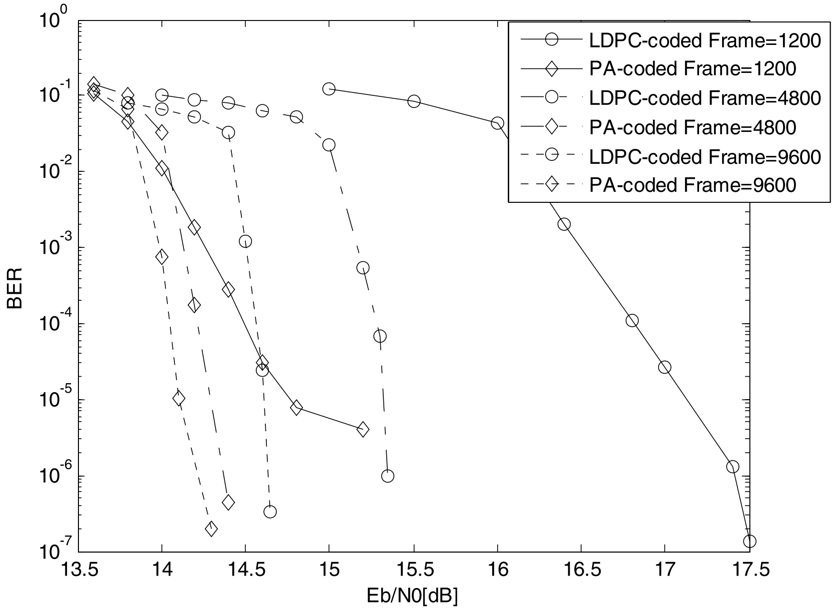


Fig. 10 BER performance of coded SIMO FM-DCSK systems with code rate of 2/3 over multipath fading channels

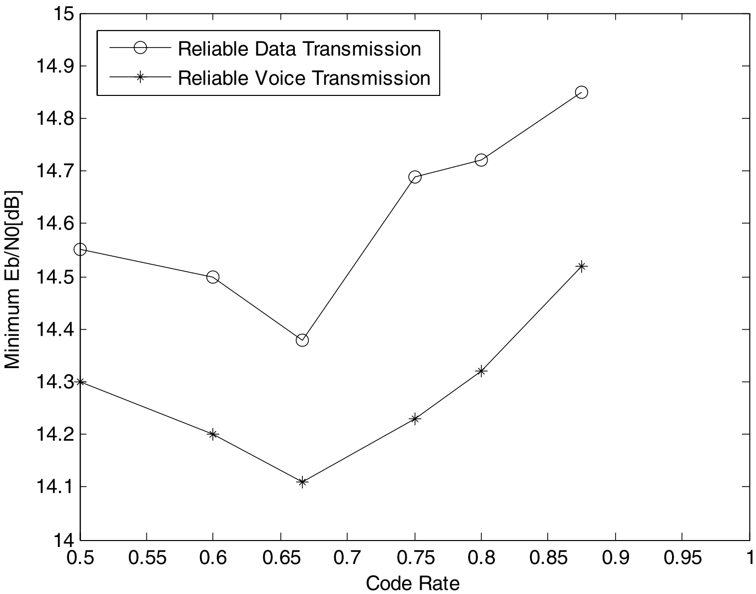


Fig. 11 Minimum required E_b/N_0 vs. code rate for the PA-coded SIMO FM-DCSK system with a fixed frame length of 4 800 bits over multipath fading channels

the PA-coded SIMO FM-DCSK system are illustrated in Fig. 11, which has a fixed frame length of 4 800 bits with different code rates of 1/2, 3/5, 2/3, 3/4, 4/5, and 7/8. It is obvious from Fig. 11 that the optimum code rate for the PA-coded SIMO FM-DCSK system should be 2/3. It is clear from Fig. 10 that the PA-coded SIMO FM-DCSK system performance has no error floors and has a 1 dB coding gain relative to that of the LDPC-coded SIMO FM-DCSK system at a BER of 10^{-6} when the code rate is 2/3 and the frame length is 4 800 bits. Hence, for the SIMO FM-DCSK systems, it is desirable to choose PA codes as the error control codes and 2/3 as the code rate along with a 4 800-bit frame length over multipath fading channels.

5 Conclusions

The BER performance advantage of a PA-coded SIMO FM-DCSK system over that of LDPC and convolutional coded systems has been investigated through simulations over multipath fading channels. The simulation results have demonstrated that PA codes can exhibit a much better error-correcting capability than the other two coding schemes in terms of contributing to the SIMO FM-DCSK scheme at medium and long frame lengths, although the former has an error floor under the condition of a short frame length of 1 200 bits. In particular, the optimum code rate for the PA-coded SIMO FM-DCSK system can be attained through the curve between the minimum required E_b/N_0 and the code rate. Moreover, it is found that the PA-coded SIMO FM-DCSK system with code rate 2/3 performs much better than the other systems with different code rates when the frame length is 4 800 bits. Thus, PA code appears to be a strong candidate as an error-correcting scheme for the SIMO FM-DCSK system due its outstanding advantages in terms of better performance, simple encoding and decoding structures, and flexibly adjustable code rate. In particular, a PA-coded system will attain its optimum performance at both medium and long frame lengths over multipath fading channels. Clearly, based on our simulation results, a code rate of 2/3 and a frame length of 4 800 bits can be chosen as a set of optimum parameters for a desired PA-coded SIMO FM-DCSK system.

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