Performance Comparison between Non-Binary LDPC Codes and Reed-Solomon Codes over Noise Bursts Channels

Junbin Chen[1]
1. Dept. of Communication Engineering, Xiamen University
Xiamen,361005,China
Email: junbin@vip.sina.com

Lin Wang[1,2]
2. ChongQing University of Posts and Telecomm
ChongQing,400065,China
Email: wanglin@xum.edu.cn

Yong Li[1]
1. Dept. of Communication Engineering, Xiamen University
Xiamen,361005,China
Email: xuehan_xx@126.com

***Abstract*— In this paper, the performance of non-binary low-density parity-check (LDPC) codes is investigated with high rates ($R > 2/3$), and small block lengths ($N < 5000$bits) over noise bursts channels, and is compared with that of Reed-Solomon (RS) codes. Meanwhile, a new optimized encoding scheme of non-binary LDPC codes is presented, and a new decoding algorithm with reduced-complexity is introduced. The simulation results indicate that the non-binary LDPC codes are very effective against noise bursts, and are about 2.7dB superior to the RS codes at a frame error rate (FER) of FER=10^{-4}, even when the burst length up to 144 bits long. It is of important value in magnetic recording systems.**

I. Introduction

The first work on non-binary LDPC codes appeared in [1], [2]. Similar to binary LDPC codes [3], [4], the non-binary LDPC codes can also be described by a low-density parity-check matrix H, but the finite field extents from $GF(2)$ to $GF(q)$. And it was shown in [1], [2], [5] that the non-binary LDPC codes with low code rates, from 1/4 to 1/2, outperform binary LDPC codes over additive white Gaussian noise (AWGN) channels. In current paper, the non-binary LDPC codes with high rates and small block length over noise bursts channels are explored.

Compared with RS codes, which are widely used as an industry standard in magnetic recording systems and communication systems to recover the burst noise, non-binary LDPC codes not only have the characteristics of non-binary codes, but also introduce the soft decision iterative decoding algorithm. So the error control system based on non-binary LDPC codes would be more powerful against burst impairments.

The remainder of the paper is organized as follows. Section II presents an optimized encoding scheme of non-binary LDPC codes. Section III introduces a new decoding algorithm with reduced-complexity. Section IV describes the burst noise model and the system diagram, and the simulation results are also provided. Section V gives the concluding remarks.

II. Optimized Encoding Scheme of Non-binary LDPC Codes

For non-binary LDPC codes, their efficacy in noise bursts can be explained via a Tanner graph representation of the code and the associated message-passing decoding algorithm [6], [7].

Fig. 1 depicts a section of a Tanner graph for some non-binary LDPC code, where some code symbols are affected by a noise burst. Now consider the decoding of the i^{th} variable node, which has been exposed to the noise burst. As described in the figure, the s^{th} and the t^{th} variable nodes are involved in the same check with the i^{th} variable node, but have not been exposed to the burst. So, associated with the message-passing decoding algorithm, the correct information from the s^{th} and the t^{th} variable nodes can be used to obtain the correct value of the i^{th} variable node via message passing (through the check k), at least after a few iterations. It can also be discussed in the same fashion if there are two or more variable nodes exposed to the burst in the same check, but more expense of the system (*eg.* more iterative times or more extrinsic information) would be needed for getting the correct values.

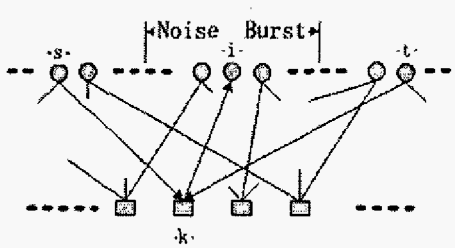

Fig. 1: A section of Tanner graph for non-binary LDPC codes

Assuming that s represents the minimum distance among the s^{th}, the i^{th}, and the t^{th} variable nodes. And T denotes the noise burst length. Thus for the check k, if $T < s$, it is effective against the noise burst with length T. So generally speaking, if in any span of s consecutive code symbols in each codeword, no two symbols are involved in the same check, the

0-7803-9015-6/05/$20.00 ©2005 IEEE.

non-binary LDPC codes will be effective against any noise bursts with length $T < s$. In terms of the parity-check matrix H, this means that any s consecutive symbols in any row contain only a single non-zero element from $GF(q)$. Since we investigate non-binary LDPC codes for improving the error correction capacity over noise bursts channels, the value of s should be maximized. Thus a new optimized encoding scheme for non-binary LDPC codes is presented.

First, a reasonable $s < N/W_r$ is chosen, where N is the length of the codeword, and W_r is the row weight of the parity-check matrix H. Then, for each column, W_c locations are randomly chosen, where W_c denotes the column weight of H, and filled with non-zero elements from $GF(q)$. Meanwhile, both the cycle-four and the s constraints are considered in the encoding algorithm. So, the generated matrix is free from cycle-four, and guarantees that, for any noise bursts with length $T < s$, there is only a single variable node exposed to the noise bursts in the same check equation.

And see Fig. 1, theoretically the maximum correctable noise burst length for a non-binary LDPC code with parameter s, should be estimated as $2s$ symbols. Thus, the non-binary LDPC codes constructed by this optimized encoding scheme should be better than the ones generated conventionally (without the constrain s).

III. Decoding Algorithm with Reduced-complexity for Non-binary LDPC Codes

The belief propagation (BP) algorithm for non-binary LDPC codes is done in exactly the same two steps as for binary LDPC codes: a row step and a column step [7].

$$r_{mn}^{f_i} = \sum_{x_n=f_i} \delta\Big(\sum_{n\in N(m)} H_{mn}x_n = z_m\Big) \prod_{n'\in N(m)\backslash n} q_{mn'}^{x_{n'}} \quad (1)$$

$$q_{mn}^{f_i} = \alpha_{mn} f_n^{f_i} \prod_{m'\in M(n)\backslash m} r_{m'n}^{f_i} \quad (2)$$

Where $N(m) = \{n' : H_{mn'} \neq 0\}$, $N(m)\backslash n = \{n' : H_{mn'} \neq 0, n' \neq n\}$; $M(n) = \{m' : H_{m'n} \neq 0\}$, $M(n)\backslash m = \{m' : H_{m'n} \neq 0, m' \neq m\}$. And x_n can be any $f_i \in GF(q)$, $i = 0, \ldots, q-1$. The posterior probabilities are computed as

$$q_n^{f_i} = \alpha_n f_n^{f_i} \prod_{m'\in M(n)} r_{m'n}^{f_i} \quad (3)$$

The hard decision is made as $\hat{x_n} = \arg\{\max(q_n^{f_i})\}$.

The computation complexity of the algorithm described above can be reduced through the idea of using a fast Fourier transform (FFT) in the BP decoding, which was proposed in [5] and [8].

Notice the summation in the row step, equation (1), that represents a convolution of the quantities $q_{mn'}^{x_{n'}}$. So, the summation can be replaced by a product of the fast Fourier transforms of $q_{mn'}^{x_{n'}}$, followed by an inverse Fourier transform. Let $(Q_{mn'}^0, \ldots, Q_{mn'}^{q-1})$ represent the Fourier transform of the vector $(q_{mn'}^0, \ldots, q_{mn'}^{q-1})$. After permuting the components to take account of the matrix entry $H_{mn'}$, now $r_{mn}^{f_i}$ is the f_i^{th} coordinate of the inverse transform of

$$\Big(\Big(\prod_{n'\in N(m)\backslash n} Q_{mn'}^0\Big), \ldots, \Big(\prod_{n'\in N(m)\backslash n} Q_{mn'}^{q-1}\Big)\Big)$$

The update of the column step, equation (2), is unchanged.

The Fourier transform over $GF(q = 2^p)$ is not a q-point FFT but a p-dimension *two*-point FFT. An example for $q = 4$ is illustrated in Fig. 2. The field elements are represented in polynomial form. Hence for $GF(4)$ we have

$$Q_{mn'}^0 = (q_{mn'}^0 + q_{mn'}^2) + (q_{mn'}^1 + q_{mn'}^3) \quad (4)$$

$$Q_{mn'}^1 = (q_{mn'}^0 - q_{mn'}^2) + (q_{mn'}^1 - q_{mn'}^3) \quad (5)$$

$$Q_{mn'}^2 = (q_{mn'}^0 + q_{mn'}^2) - (q_{mn'}^1 + q_{mn'}^3) \quad (6)$$

$$Q_{mn'}^3 = (q_{mn'}^0 - q_{mn'}^2) - (q_{mn'}^1 - q_{mn'}^3) \quad (7)$$

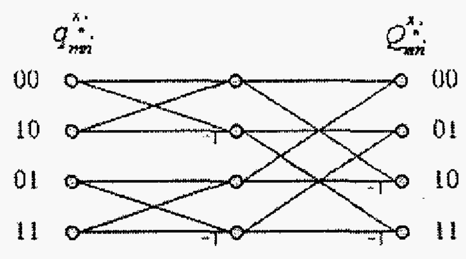

Fig. 2: FFT for $q = 4$

IV. Performance of Non-binary LDPC Codes

The following three codes are investigated in this paper. 1) Non-binary LDPC code (1182, 1088), which is denoted by LDPC-I, with $q = 16$, $W_c = 3$, $s = 20$, and the code rate $R = 0.9205$. 2) Non-binary LDPC code (1152, 1024), which is denoted by LDPC-II, with $q = 16$, $W_c = 3$, $s = 35$, and $R = 0.8889$. 3) RS code (511, 461) over $GF(2^9)$, with $R = 0.902$. These three codes are investigated in the similar code length (in bit) and the same environment of channels.

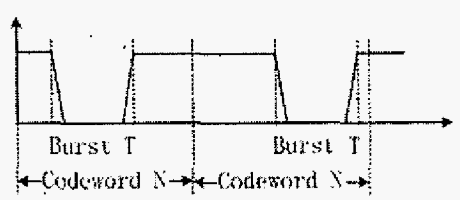

Fig. 3: Burst noise model

The burst noise model is shown in Fig. 3. We assume that one fixed length-T-symbols burst with random starting location occurs in every length-N-symbols transmitted codeword.

Furthermore, we assume that the envelope of the burst is rectangular.

The system diagram under investigation is shown in Fig. 4. Assume the ability to detect the presence of a noise burst, a fairly simple matter in practice. And the channel state information (CSI) is available for both the channel detector and the decoder. CSI is used to set the code symbols to zero, which were received during the noise bursts.

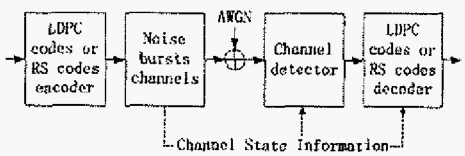

Fig. 4: System diagram

Fig. 5 presents the results for the non-binary LDPC codes and the RS code over AWGN channels. Also shown is the performance of a binary LDPC code (labeled as B-LDPC) of column weight three, which has the same code length (in bit) and code rate as LDPC-II. The curve for the RS code is inferior to that of the LDPC-I and LDPC-II codes by about 1.5dB and 2.4dB respectively. And the LDPC-II code also performs 0.2dB better than the B-LDPC code at a bit error rate (BER) of BER=10^{-5}

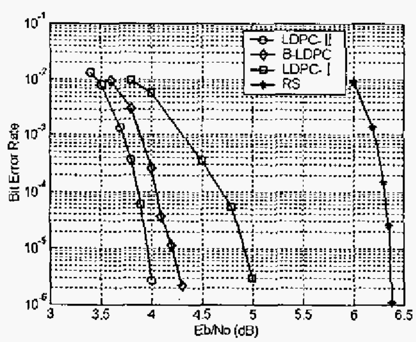

Fig. 5: Performance of LDPC codes and RS codes over AWGN channels

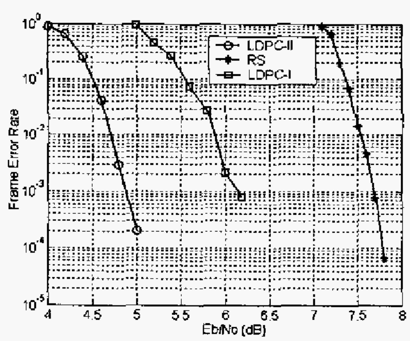

Fig. 6: Performance of non-binary LDPC codes and RS codes over noise bursts channels with burst-length 144 bits

Fig. 6 presents the frame error rate curves for the non-binary LDPC codes and the RS code over a noise bursts channel with burst-length 144bits. The curve of the LDPC-II code is roughly 2.7dB superior to that of the RS code (at FER=10^{-4}), whereas the performance of LDPC-I code is about 1.6dB better (at FER=10^{-3}) than that of the RS code. It is obvious that over both AWGN channels and noise bursts channels, non-binary LDPC codes perform better than RS codes, even when the code rates of non-binary LDPC codes are higher than that of RS codes.

The system with the LDPC-II code is also simulated at $E_b/N_0 = 7$dB, and the performance is shown in Table I. The frame error rate is the ratio of error frames to total simulation frames. Roughly, this system is able to correct noise bursts of length up to 260bits, which is approximately consistent with the theoretical maximum correctable burst length discussed at the end of Section II, namely, $2s$ symbols, $2 \times 35 symbol \times 4 bit/symbol = 280$bits. And from Fig. 7, it can also be seen that the LDPC-II code constructed by the optimized encoding scheme perform roughly 0.1dB better than the ones constructed conventionally (without the s constrain) [5], over noise bursts channels with burst-length 144bits.

TABLE I: Frame error rate with noise bursts, LDPC-II

Length of Defect (bit)	Frame Error Rate ($E_b/N_0 = 7$dB)
80	0/10000
140	0/10000
200	0/10000
260	0/10000
280	1/10000
300	19/10000
320	129/10000

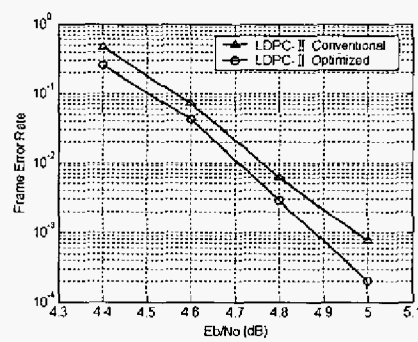

Fig. 7: Optimized encoding scheme *Vs* Conventional encoding scheme

In summary, non-binary LDPC codes beat RS codes over both AWGN channels and noise bursts channels, because they not only have good distance properties and the characteristics of non-binary codes, they also have a soft iterative decoding algorithm that can (almost always) correct errors at noise levels greater than those that would be tolerated by any bounded-distance decoder. For practical purposes, RS codes only have bounded-distance decoders. Furthermore, non-binary LDPC codes can be further optimized via employing the optimized encoding scheme. Meanwhile, non-binary LDPC codes outperform binary LDPC codes over AWGN channels, even with the code rate high up to 0.8889.

V. Conclusion

Non-binary LDPC codes provide a new error control technology of combining the non-binary codes and the soft iterative decoding algorithm. The simulation results also demonstrate that non-binary LDPC codes are well suited for recovering burst noise and, in fact, perform about 2.7dB better than RS codes (at FER=10^{-4}). Moreover, the performance of non-binary LDPC codes can be further improved via introducing the optimized encoding scheme, which is discussed in Section II. Based on these results, it can be foreseen that non-binary LDPC codes could be a good alternative to RS codes, and it could be of important value in magnetic recording system or other communication systems under severe burst noise environments. In our future work, we will focus on analyzing and improving the performance of non-binary LDPC codes on partial-response channels.

Acknowledgment

The authors would like to thank Mr. Weikai Xu for his useful help about the simulation sincerely. This work is sponsored by Chinese national nature science fund project (60272005). And the corresponder is wanglin@xmu.edu.cn.

References

[1] M. C. Davey and D. J. C. MacKay, "Low-density parity check codes over GF(q)" *IEEE Commun. Lett.*, vol. 2, pp. 165-167, June 1998.

[2] M. C. Davey and D. J. C. MacKay, "Low density parity check codes over GF(q)," in Proc. *IEEE Inform. Theory Workshop* 1998, pp. 70-71, 22-26th June 1998, Killarney, Ireland.

[3] D. J. C. MacKay and R. M. Neal, "Near shannon limit performance of low density parity check codes," *IEEE Electronics Lett.*, vol. 32, pp. 1645, Aug. 1996.

[4] Wang Lin, Xiao Juan, Guanrong Chen, "Density evolution method and threshold decision for irregular LDPC codes," in Proc. *IEEE ICCCAS 2004*, vol. 1, pp. 25-28, 27-29th June 2004, ChengDu, China.

[5] M. C. Davey, "Error-correction using low-density parity-check codes," Ph.D. dissertation, Univ. Cambridge, Cambridge, U.K., Dec. 1999.

[6] R. M. Tanner, "A recursive approach to low complexity codes," *IEEE Trans. Inform. Theory*, vol. 27, pp. 533-547, Sep. 1981.

[7] D. J. C. MacKay, "Good error-correcting codes based on very sparse matrices," *IEEE Trans. Inform. Theory*, vol. 45, pp. 399-431, Mar. 1999.

[8] T. J. Richardson and R. L. Urbanke, "The capacity of low-density parity-check codes under message-passing decoding," *IEEE Trans. Inform. Theory*, vol. 47, pp. 599-618, Feb. 2001.